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A Genetic Model of Adaptive Economic Behavior

by

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Abstract

A model of adaptive economic behavior is developed and analyzed. The model, by relying on specific biological concepts, reduces the ambiguity which has confounded previous biological-based models, and has sufficient structure to be explicitly explored. The basic assumptions of the model are examined, and they appear to accord well with current economic and psychological theories of behavior. Furthermore, the theoretical properties of the model are derived based on recent work in computer science. These properties indicate that the basic model has a strong optimization component, which suggests that the optimization and adaptive approaches are not mutually exclusive theoretical milieus. The model is applied to a variety of economic problems, and its implications are derived and analyzed.

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1. Introduction

Under the current optimization paradigm, a rather eloquent and elaborate theory of economic behavior has been formulated. Various aspects of this formulation have been criticized by a number of economists (for example, Simon, 1955, 1959; Koopmans, 1957; Cyert and March, 1963; Winter, 1964, 1971, 1975; Nelson and Winter, 1982). Most of these criticisms have been directed at the current paradigm's reliance on equilibrium conditions and its rather extreme assumptions about the abilities of individual agents. One class of models which has attempted to circumvent these criticisms is derived from basic biological theories. This paper develops, formalizes, and applies a biologically-based model of economic behavior.

The model developed in this paper relies on two major premises: that some economic behavior has close analogues to the biological behavior of ecosystems and species, and that this behavior can be effectively modeled and analyzed. The first premise, as discussed below, has long been recognized by economists. The second premise has been more problematic, however, recent work in computer science has produced effective modeling techniques. Unlike previous biologically-based economic models, the model developed in this paper uses specific biological processes. By relying on specific processes the model can incorporate important elements from both economic and psychological theories of behavior. The use of specific processes also imposes formal structure on the model, which allows some interesting and important properties of the model to be theoretically derived.

This paper is organized into four sections. The first section provides a general discussion on the use of biologically-based models in economics, both historically and vis-a-vis the current optimization paradigm. The second section introduces a general framework for adaptive models, and discusses the problems which face all adaptive models. The third section introduces the actual model and its theoretical properties, and provides the rationale for the use of such a model in an economic context. Finally, the last section applies the model to a variety of interesting economic examples.

2. Biologically-Based Models In Economics

Biological models are well established in the history of economic thought. Boulding (1981) argued that an implicit biological model is very strong throughout most of early economic thought beginning with Adam Smith's *Wealth of Nations* in 1776. Certainly, Malthus's model of the dynamic interaction between population size and wage rates presented in *Principles of Population* (1798) is an example of an early evolutionary theory applied to an economic phenomenon.[1] The thread of biologically-based models continues up to, and is apparent in, the work of Marshall (1890), although it was around this time that an emphasis on equilibrium states started to take hold (for example, Walras, 1874). Equilibrium analysis, with rare exceptions (Schumpeter, 1912, 1942), became the dominant mode of economic reasoning.

While equilibrium models have dominated current thought, a number of recent authors have proposed biologically-based models. Alchian (1950), Friedman (1953), Winter (1964), Day and Groves (1975), Boulding (1978, 1981), Nelson and Winter (1982), and Cross, (1983), have all suggested that various biological concepts may be useful adjuncts to more traditional economic theories. For the most part, these biological models have relied upon rather general results from evolutionary and ecological biology. Only Nelson and Winter (1982) and Cross (1983) have presented well formalized and relatively testable models. All of the work indicates that economic analogues of biological concepts may prove to be fertile areas for investigation.

Economic models based on evolutionary and ecological theories provide a number of advantages over standard optimization models. The first advantage is that they place small demands on the abilities of economic agents. As pointed out by Simon (1955, 1959), psychological learning studies indicate that the abilities of economic entities to acquire and process information may be far less than that required by most optimization models. Another advantage of biologically-based models is their inherent dynamic nature. Neoclassical models, being based upon equilibrium concepts, do not operate effectively in disequilibrium states. However, a large part of interesting economic behavior occurs in a dynamic disequilibrium context. Evolutionary models, being directly based upon dynamic processes, avoid any difficulties associated with disequilibrium states. The fact that the same model applies to both equilibrium and disequilibrium conditions

[1] In fact, it was this work of Malthus's which proved to be the catalyst for Darwin's theory of evolution, C. Darwin, Autobiography, in *The Life and Letters of Charles Darwin*, Vol. I, F. Darwin (ed.) (London: John Murray, 1887).

provides a powerful technique for dealing with many problems which have eluded more orthodox solutions, including: short-run versus long-run behavior, intertemporal choices, and decisions under uncertainty.

Biologically-based models are no panacea. While they may be based on more realistic assumptions, they often lack the analytical solutions that more traditional neoclassical models provide. Although certain properties of the models can be theoretically derived, their inherent stochastic nature requires the use of simulation for more specific results. However, even in these cases the value of adaptive models must not be overlooked. Adaptive models may suggest general results about how rapidly, and under what conditions, behaviors will tend towards optimal patterns. Thus, they could provide strong support for the application of more traditional neoclassical models to certain situations.

The model of economic behavior developed in this paper is closely related to models used in evolutionary biology. Most of the biologically-based economic models have relied on relatively general results from a limited set of biological theories. Unlike these models, the model pursued here, based on an artificial genetic algorithm created by Holland (1975, 1980), relies not only on broad biological notions, but also incorporates close analogies to more specific biological processes (for example, the recombination of genetic material). By incorporating specific processes a highly efficient adaptive model can be developed. The use of specific processes also allows close analogies to be drawn between the model and current theories of economic and psychological behavior. Before presenting the actual model, a general understanding of the context in which such models operate is in order. Such an understanding not only illuminates the necessary elements of any adaptive model, but it also clarifies the types of problems which such models must surmount.

3. Adaptive Systems[2]

In order to develop a model of adaptive economic behavior one must first define the overall system in which the adaptation takes place. By defining the general adaptive system, the salient elements of the adaptive process can be illuminated, and their properties can be formally examined. Once this is done, problems which confront any adaptive model will be clarified. From this knowledge an effective adaptive model can be developed.

In general, any system with an adaptive process can be described in terms of three major elements: allowable structures, an environment, and an adaptive plan. Individual actors (for example, a consumer, a firm, an individual organism) interact with the environment through various characteristics described by structural forms (for example, a purchasing pattern, decision rules, genetic and morphological forms). At every time period [3] an environment determines the performance of the various structures, and provides a given level of information back to the actor. No assumptions are made about the stability of the environment, or about the level of information presented to the actor—although some degree of feedback is required. Finally, dynamic interactions between the environment and the structural forms are determined by the adaptive plan. Adaptive plans determine changes in the existing set of structures, perhaps stochastically, from one time period to the next based on the available information.

More formally, any adaptive system can be defined by:

$A = \{A_1, \ldots, A_N\}$, the set of possible structures;

$E = \{E_1, \ldots, E_E\}$, the set of possible environments;

$\tau =$ an adaptive plan operating in discrete units of time indexed by t, where $\tau : A \times I(t) \to \Omega$;

$I(t) =$ the information available to the adaptive plan from the environment at time t;

$\Omega = \{\omega_1, \ldots, \omega_n\}$, the set of operators such that $\omega_i : A \to P$;

$P =$ a probability density function over A;

The interpretation of this formal system is as follows: at any time, t, the adaptive plan tests a subset of structures, $A(t)$, against the environment. The environment provides information, $I(t)$, about the per-

[2] The basic elements of this and the next section rely heavily on the work of Holland (1975).

[3] For simplicity discrete time is usually assumed.

formance of $A(t)$, and from this the adaptive plan develops a new set of structures: $\tau(A(t), I(t)) \rightarrow \omega_t$, and $\omega_t(A(t)) \rightarrow P(t+1)$, which will result in $A(t+1)$.

Given the above system, one only has to be able to define the environment, structures and the adaptive plan to use an adaptive model. However, this is not a simple task. Decisions must be made about which elements of structure—and consequently which elements of the environment—are important to the analysis. Implicit in this decision is the choice of a feedback and information mechanism in the environment. A number of difficulties also arise in the choice of an adaptive plan. The ideal adaptive plan must not only overcome numerous mathematical obstacles in developing well adapted structures, but it must also meet a number of other criteria (such as, operating under conditions of limited information, computational ability, and time in the case of an economic model).

While most of the difficulties in defining the environment and structures can usually be overcome, the choice of an appropriate adaptive plan is still problematic. Usually, the important elements of the environment and structures will become apparent after the context for the model has been decided. Unfortunately, the choice of an adaptive plan is not as well delineated. In general, regardless of the context, to develop well adapted structures adaptive plans must surmount the following mathematical difficulties: complicated structures, non-linear performance measures, and the trade-off between exploitation and exploration.

The structures which an adaptive plan must manipulate can present many difficulties. One major problem is that an enormous number of structures can confront an adaptive plan. This may be true because of a large variety of choice may be available, for example, the amount of money an individual should spend on education, or because of the explosive combinatorics facing even relatively simple situations, consider the choice of a strategy which determines whether to invite friends over for a dinner based upon their weekly invitations over the past 5 weeks, this results in 2^{32} or approximately 4 billion potential strategies. The fact that an enormous number of structures may face an adaptive plan eliminates the potential usefulness of an enumerative approach. Furthermore, even when the number of structures is small, it may be difficult to define the exact elements of any given structure which are responsible for different performance levels. Thus, while it may be easily determined that a given structure performs well in a given situation, it may be difficult to determine which specific element of the structure, or group of interrelated elements are responsible for this performance.

To produce well adapted structures adaptive plans must rely on various performance measures derived from information available in the environment. Unfortunately, performance measures may not be the result of nicely behaved functions. These functions may exhibit a number of undesirable properties, including: non-linearity (many local optima), high dimensionality, and discontinuity. Each of these elements complicates the task of efficiently developing high performance structures.

Adaptive plans must also effectively manage the exploration of new structures and the exploitation of existing structures. A number of untried structures may contain the key elements necessary for improved performance. However, if only untried structures were used by the adaptive plan, present performance would be sacrificed by ignoring the best of the already explored structures. Thus, adaptive plans must find efficient ways to simultaneously explore new structures and exploit existing structures.

Finally, some other criteria must be imposed on the adaptive plans if they are to offer a reasonable alternative to existing economic optimization models. Desirable plans must not have excessive information, storage, or manipulation requirements, and should accord well with current psychological theories of learning behavior. They should also perform well over a variety of environments. Furthermore, if the adaptive plans are to serve as a hypotheses about behavior over finite time periods, and if they are to offer support for neoclassical outcomes, they should converge relatively rapidly to high performance levels.

Holland (1975) proposed an adaptive plan which meets the above requirements. Utilizing analogues to basic biological and genetic systems, Holland has developed a highly efficient adaptive plan in the form of a genetic algorithm.[4] The genetic algorithm overcomes the aforementioned problems by allowing each interaction between the environment and an individual structure to represent a sample observation on a relatively large number of structural forms, and from this information develop new structures which have a high probability of good performance.

[4] A description of the actual algorithm is presented in the Appendix.

4. The Adaptive Model

The adaptive model presented here assumes that economic agents follow an adaptive plan which modifies a small set of remembered behaviors based on the relative performance of each behavior. The model assumes that behaviors are composed of individual building blocks, and that individuals reproduce and recombine the building blocks of successful actions. The way in which new behaviors are developed is directly analogous to some specific biological processes. By basing the model on specific processes, implications about the outcomes of such behavior can be formally derived and modeled. Although the adaptive plan is grounded in biology, strong support exists for the use of such an approach as a model of cognitive economic behavior.

The model assumes that every behavior, i, of an agent can be represented by an l-tuple

$$A_i = (a_{i1}, \ldots, a_{ij}, \ldots, a_{il}).$$

Each $\{a_{ij}\}$ can take on one of m_j *allele* values at every *locus* j. Individual allele values at a given locus, or values from a subset of loci, determine the actual structural attributes of A_i. This system allows many structures to be easily specified, for example, suppose a_{ij} has m different values for each allele, then the above system will allow m^l different structures to be defined. [5] While this technique of defining behaviors may appear rather extreme, it does have a natural interpretation: that any behavior is composed of individual building blocks.

Any time a behavior is undertaken the agent receives a known payoff. The function, $\mu_t : A \rightarrow \Re$, gives the amount of the payoff, where the form of μ_t depends on the environment existing at time t. Note that the payoff is contingent on the existing environment which may change over time, and that the only information provided to the agent is the payoff for each attempted behavior. Although a large variety of potential payoff functions exist, some particularly convenient forms for economic applications are standard utility and profit functions.

At every time period, t, agent's maintain (remember) a small subset of behaviors, $A(t)$. From this subset the agent develops new behaviors through the application of two basic, but powerful, ideas. The first idea is reproduction based on performance, whereby those actions which have higher payoffs will be reproduced (remembered) more often. The second notion is that new actions will be developed by recombining the building blocks of existing actions. Through the incorporation of these two ideas the model effectively and easily develops new, high performing structures.

By reproducing actions based on their payoffs continual improvement in the maintained set of actions is encouraged. Suppose that the expected number of offspring is proportional to performance, that is,

$$N(A', t+1) = \frac{\mu_t(A')}{\sum_{A \in A(t)} \mu_t(A)/M}, \tag{1}$$

where $N(A', t+1)$ is the number of $A' \in A(t)$ at time $t+1$, and M is the size of the population $A(t)$. This equation implies that those structures which do better than average will increase over time, and that those which do worse than average will decline. Furthermore, note that if the environment is unchanging the average performance of the population will increase, putting increased survival pressure on existing behaviors. Under a system of reproduction by performance, existing behaviors which perform well perpetuate, however no new behaviors are introduced.

A powerful way to introduce new behaviors into the system is through the recombination of those structures chosen for reproduction. Direct analogues to basic genetic operators easily and efficiently recombine existing structures. One of the most important genetic operators is cross-over. The cross-over operator works as follows: two structures are chosen from the population and a cross-over point, x, is randomly chosen between two of the loci ($x \in [1, l-1]$), two new structures are formed by combining the alleles from the first x loci on the first structure with the alleles located at the $x+1$ to the l loci on the second structure, and combining the first x alleles on the second structure with the $x+1$ to l alleles on the first structure. The second commonly used genetic operator is mutation. Mutation randomly changes the allele value at a given

[5] One structural representation which turns out to be convenient for many applications is the binary form, where $a_{ij} = \{0, 1\} \quad \forall\, i, j$.

locus. By using cross-over and mutation the existing structures are recombined in a surprisingly effective manner. [6]

The combination of reproduction based on performance and recombination via genetic operators results in a very efficient sampling plan over the space of behaviors. This occurs because although the agent only explicitly receives information about individual behaviors, he is implicitly obtaining information about the performance of important building blocks of all actions. This information is compactly stored in the existing set of structures, and is effectively used in the formation of new structures. Thus, not only does the sampling plan efficiently sample the space, but it does so with very low information and processing requirements.

By defining behaviors as l-tuples various structural forms, called schemata, can be easily derived. A given schema, $\xi \in \Xi$, is defined by specifying those loci of a structure which must exactly match a predetermined allele value, and consequently those loci which may have any value. For example, if the structures we are trying to represent are meals consisting of either soup or salad for an appetizer, and either hamburgers, hot dogs, or quiche for a main course (implying a potential of six structures), then one potential schema would be those meals with soup (admitting three structures into the schema), and another schema would be meals with quiche as a main course (matching two structures). From the definition of a structure above, one can show that there exist $\Pi_{j=1}^{l}(m_j + 1)$ schemata over the set of all structures, A. For example, a binary structure with $l = 8$ has 3^8 or 6561 schemata over its 2^8 or 256 potential structures.

Schemata are of paramount importance to the performance of the adaptive model. First, note that any single behavior, A_i, represents 2^l different schemata, since any given allele of A_i can represent either a schema with $a_j^s = a_{ij}$ (where a_j^s is the value of the schema at locus j), or a schema which does not care about the value at that locus. Thus, by testing a single structure against the environment the adaptive plan not only obtains information on the performance of that particular structure, but it also obtains information on many potentially useful schemata (in the example of the 8 bit binary structure, a single structure contains about 4 percent $(2^8/3^8)$ of all potential schemata even though it represents only about .25 percent (1/256) of all potential structures. [7]).

By using reproduction and recombination, the adaptive model effectively samples the schema space. Each test of a behavior in the environment results in payoff information over all of the schemata contained in the action. With this information the adaptive plan first reproduces those schemata which exhibit good performance. Then, by using genetic operators, the adaptive plan combines these old schemata with new schemata and forms new structures which can then be tested. This allows the plan to exploit information gained about the structures previously tested while simultaneously exploring new structures.

If structures are only reproduced according to performance, from (1) the expected number of schema of type ξ represented at time $t+1$ will be

$$N(\xi, t+1) = \frac{\sum_{A^* \in A(t) \ni \xi} \mu_t(A^*)}{\sum_{A \in A(t)} \mu_t(A)/M} \qquad \forall\ \xi, t.$$

Multiplying and dividing the right hand side by $N(\xi, t)$ yields

$$N(\xi, t+1) = \frac{\hat{\mu}_t^{\xi}}{\hat{\mu}_t} N(\xi, t) \qquad \forall\ \xi, t, \tag{2}$$

where $\hat{\mu}_t^{\xi}$ is the average performance of ξ in the sample $A(t)$, and $\hat{\mu}_t$ is the average performance of all of the behaviors in $A(t)$. Equation (2) implies that with reproduction alone schemata in the original subpopulation, $A(0)$, which belong to structures performing better than average will increase in the population. However,

[6] Cross-over and mutation are only two of a large class of recognized genetic operators. While other operators have been used with some success in other applications, the two used here have been shown to be highly effective and are sufficient for the basic model.

[7] Note that to represent the most schemata per structure it is better to define structures with a small number of possible values per allele. For example, a structure $A_i = (a_{i1}, a_{i2})$ with $a_{ij} = 1, \ldots, 16$ for $j = 1, 2$ will also define 256 structures, but will only represent 4 schema per trial (or, about 1.38 percent of all potential schemata).

reproduction alone does not admit any new structures into $A(t)$, and hence no new schemata are tested and the sample of old schemata is biased towards those structural arrangements which existed in the original population.

In order to develop new schemata and to test old schemata in different structural contexts, the cross-over operator can be employed. The cross-over operator serves two major functions: it creates instances of new schemata while simultaneously maintaining old schemata, and it links schemata which are physically close together. Holland (1975, p. 99) showed that if the two originally chosen structures differ at L positions to the left of and at R positions to the right of the cross-over point then either one of the newly formed structures will contain $2^l - 2^{l-L} - 2^{l-R} + 2^{l-(L+R)}$ new schemata (schemata not found in either of the two original structures) and $2^{l-L} + 2^{l-R} - 2^{l-(L+R)}$ old schemata (schemata already found in the two original structures). The second function of the cross-over operator is to encourage linkage of schemata. If one looks at only the defining positions of a schema (that is, those positions which specify a specific allele value) then the probability of schema ξ being broken by a randomly chosen cross-over point is equal to $(l(\xi)-1)/(l-1)$ where $l(\xi)$ is the smallest number of loci which contain all of the defining alleles for ξ. Since $l-1$ is constant for all structures, schemata which are closer together have a smaller probability of being separated during cross-over.

If the cross-over operation is introduced into the adaptive plan the expected number of schemata of type ξ that will be present at time $t+1$ can be obtained by combining (2) with the lower bound[8] of the probability of cross-over not breaking up a schema (from above):

$$\mathrm{E}(N(\xi, t+1)) \geq \frac{\hat{\mu}_t^\xi}{\hat{\mu}_t}[1 - \frac{(l(\xi)-1)}{(l-1)} P_c] N(\xi, t) \qquad \forall\ \xi, t, \tag{3}$$

where P_c is the probability of cross-over. This implies that a schema, ξ, will increase its representation in a population as long as

$$\hat{\mu}_t^\xi \geq \frac{1}{[1 - P_c(l(\xi)-1)/(l-1))]} \hat{\mu}_t \qquad \forall\ \xi, t.$$

Note that the above equation indicates that longer schemata must have better relative performance than shorter schemata to increase in the population. Also notice that each schema increases or decreases in size independent of the dynamics of the other schemata in the population.

The fact that the cross-over operator induces a linkage on alleles which are close together may be problematic if sets of alleles which induce high performance are separated by large distances on the structure. In this case the use of only the cross-over operator may discourage the discovery of potentially important schemata. One way to circumvent this difficulty is to introduce an inversion operator. The inversion operator randomly selects two breakpoints on the structure, and reverses the ordering of the alleles between these two points. This allows alleles to be relocated, perhaps linking previously distant but closely related sets. Experiments with inversion operators indicate that they may be helpful in initial stages of adaptation, but contribute little in latter stages (Frantz, 1972). To avoid unnecessary complexity, the inversion operator has not been used.

Finally, to prevent the adaptive plan from getting trapped by eliminating initially poor performing, but ultimately important schemata, a mutation operator is allowed. The mutation operator randomly changes existing alleles on the structure. By doing so, this operator prevents the elimination of potentially valuable schemata. Mutation's value is not in generating new structures to test, since this is equivalent to using an enumerative approach, but rather in the prevention of entrapment on false peaks. Thus, only a small probability of mutation is required. Equation (3) can be modified to reflect the influence of mutation on the sampling structure of the schemata space,

$$\mathrm{E}(N(\xi, t+1)) \geq \frac{\hat{\mu}_t^\xi}{\hat{\mu}_t}[1 - \frac{(l(\xi)-1)}{(l-1)} P_c](1-P_m)^{d(\xi)} N(\xi, t) \qquad \forall\ \xi, t$$

where P_m is the probability of mutation and $d(\xi)$ is the number of defining elements in ξ. All of the previous results about schemata growth hold with the mutation operator.

[8] This is a lower bound since it is possible, if both parents have the appropriate alleles, for a schema to remain intact even if the cross-over point is in the middle of the schema.

The incorporation of the above genetic operators along with reproduction by performance results in a powerful adaptive plan. Schemata increase or decrease based only upon their own observed performances. Holland (1975, pp. 121–40) demonstrated that the rate at which schemata are sampled closely corresponds to the optimal sampling path in the canonical n-armed bandit problem, regardless of the form of μ_t. Moreover, while the adaptive plan is generating an appropriate sampling plan for the existing schemata it is simultaneously generating new schemata and structures to test in a way which encourages high interim performance levels. The plan accomplishes the above, while avoiding entrapment on false peaks.

The performance of this adaptive plan has been extensively studied. Frantz (1972) showed that the genetic algorithm effectively adapted to highly non-linear systems. Martin (1973) investigated the asymptotic properties of a similar class of adaptive plans. She found that under certain restrictions the adaptive plan does converge to a set of "good" strategies. DeJong (1975) simulated various versions of the algorithm over a variety of environments including: continuous, discontinuous, unimodal, multimodal, convex, nonconvex, low-dimensional, high-dimensional, and noisy functions. His results, later corrected by Bethke (1981), indicated that the genetic algorithm performed better than commonly used function optimization techniques. DeJong found that the algorithm exhibited rapid initial improvement, but that it usually converged towards a point near, but not on, the optimal value. Subsequent analysis indicated that this was caused by the phenomenon of genetic drift, where certain important alleles are lost initially and not recovered by the population due to purely stochastic effects. DeJong's analysis also included an exploration of appropriate parameter settings. In general these results found that: (1) larger remembered population sizes produce better long term performance but slower initial adaptation, (2) larger mutation rates prevented allele loss and provided better initial but poorer long term performance, and (3) larger cross-over rates prevent allele loss but slowed initial performance. DeJong's investigation showed that the version of the algorithm presented here, along with the parameter settings used in the following simulations, performed well across a variety of environments. Finally, Bethke (1981) extended DeJong's results. Through the use of Walsh transforms he demonstrated that those functional forms which confounded the genetic algorithm were also the forms which provided the most difficulty for other commonly used function optimization methods.

To utilize the above adaptive model one must assume the following: (1) behaviors which interact with the environment can be described as a set of alleles, $\{a_{i1}, \ldots, a_{il}\}$, (2) a single performance measure conditional on the environment exists for any behavior, and (3) systems using the behaviors have: (a) the ability to remember a limited set of past behaviors and their corresponding performance levels, and (b) the ability to do simple statistical calculations, perform simple genetic operations, and thereby implement the adaptive plan. While the first assumption does force a fair amount of rigor on allowable activities, it also allows a large amount of flexibility in defining behaviors. The assumption of a single performance level being generated by the environment is consistent with much of the theory present in economics and the other social sciences. Moreover, since payoff functions may give far more information, the single payoff measure used in the model serves as a useful lower bound. Finally, the assumptions about the abilities of the system may or may not hold depending upon the actual system being modeled.

The adaptive model presented here could be applied to either micro- or macro-level systems. At the micro-level, the model could be applied to the behavior of an individual decision maker. While it is doubtful that individuals actually use a formal version of the genetic algorithm in their decision calculus, informal versions of the model are well within the cognitive bounds of decision makers and accord well with some current theories of psychological behavior. The plausibility of the model may be enhanced in macro-level applications, where the assumption of various market mechanisms may be sufficient to drive the model.

In order to apply the model to micro-level applications, one must assume that individuals implement a version of the genetic algorithm. Intuitively, the basic ideas of the model seem rather plausible, that is, that individual's repeat behaviors with favorable outcomes, perhaps combining them with elements of other favorable behaviors. [9] In fact, some support for the behavioral basis of the genetic algorithm exists in the

[9] The formal model implies a number of potential experiments which would confirm its applicability to micro-level behavior. One implication of the model is that the outcomes of repeated decisions should converge towards optimal behavior. The model also implies that decision-makers remember and use a base set of structures, therefore, studies attempting to delineate these structures would be enlightening. The model also gives one some idea about how new structures are formed, thus one test would be to impose the structural foundations on a given situation, and observe behavior over time. For example, individuals could be allowed to purchase a market basket of goods at a supermarket every week, and their purchases could

psychological literature. Models of associative learning (Estes, 1954, 1961, 1973, 1976) closely correspond to the above model. Associative learning models assume that behaviors, and their subsequent payoffs, are associated with a sample of the set of characteristics occurring during the event. This notion is very similar to the model here, where the payoffs to a given structure (behavior) are associated with the schemata in the structure.

Macro-level applications of the model can be predicated on less extreme assumptions about the psychological behavior of individual decision makers. At the macro-level, each remembered behavior in the model, $A_i \in A(t)$, can be viewed as the behavior of an individual agent in a market, community, etc. If there exist mechanisms in the system which reward individual agents for good performance, provide information about the other agent's behaviors and payoffs, and allow the imitation of other's behaviors, then the genetic algorithm is a reasonable model of such a system. For example, in the case of a competitive market, firms which perform poorly will go bankrupt and be eliminated from the population, while the more successful firms will be the focus of imitative efforts by competitors and new entrants. These two conditions, which have strong empirical support, allow the model to be implemented.

The adaptive model presented here offers numerous advantages over earlier models of adaptive economic behavior. Although one of the primary motivations for developing adaptive models was to alleviate the reliance on excessive cognitive abilities, many of the adaptive models which were formulated still required large amounts of either memory or computation. Unlike these models, the model presented here has very low knowledge and memory requirements: agent's only have to remember a small number of behaviors and their associated performances. Another advantage of the genetic model is its ability to generalize from past performances and incorporate valuable elements of important behaviors into new behaviors. Finally, the theoretical properties of the model, along with its extensive empirical analysis, allow a good understanding of the model's behavior.

5. Applications of the Model

In order to demonstrate the usefulness of the adaptive model described above a variety of applications will be given. The applications cover a broad range of economic topics, including: consumer demand behavior, uncertainty, market structure, technological change, and economic demography. The examples were chosen for a variety of reasons: some allow a direct comparison between the adaptive approach and more orthodox approaches, others clarify various theoretical and practical issues with the model, and a few of the examples demonstrate the use of the model in situations for which more orthodox solutions are either cumbersome or do not exist. All of the examples indicate the value of the approach.

All of the applications were structured in such a way that they could be simulated using a computer.[10] Simulation is an ideal way to explore the implications of the model. Given the stochastic nature of the model, direct analytical statements about the trajectory of the model through the space of behaviors are not obtainable. However, useful probabilistic statements can be formulated. By repeatedly simulating the problem, the sensitivity of the model to initial conditions and nature can be explored, and general notions about the likely direction and outcomes of the adaptive system can be obtained.

5.1 An Application to Consumer Demand Behavior

The first application is a standard consumer demand problem. This micro-level application allows a direct comparison between the adaptive model and a well known optimization model. It also provides a good illustration of how the adaptive framework can be applied to a standard economic problem. The basic problem is as follows: a consumer is allowed to purchase two goods, x and y, which sell for market prices \$1 and $\$p_y$ respectively. The consumer is constrained by a total income of $\$I$ and has a utility function $U(x, y)$ which measures the consumer's well being up to a monotonic transformation.

The optimization approach assumes that the consumer is able to solve the following:

$$\max_{x,y} U(x,y) \quad \text{s.t. } x + p_y y = I.$$

be monitored. From this data, one could look at how the composition of the bundle changes, looking for indications of cross-over and mutation behavior.

[10] The algorithm used and the technical details may be found in the Appendix.

To solve such a problem the consumer must have full knowledge of the form of $U(x, y)$ for all values of x and y, and be able to effectively carry out some form of an optimization technique.

In order for an adaptive approach to be consistent with goal maximizing behavior, some assumptions used in the optimization approach must not hold. For example, one could assume that although consumers have well defined utility functions, they are only able to accurately evaluate their utility around bundles which they have experienced. Another possible break down in the optimization approach is that consumers do not have the requisite ability to directly maximize the known functions.

The adaptive model is constructed in the following manner. A consumer's behavior is described by a vector $\{x, y\}$ where x and y are the respective amounts of goods x and y to purchase. The utility of every behavior is given by the function $U(x, y) = u(x, y) - \Gamma(x, y)$, where $u(x, y)$ is a standard utility function and $\Gamma(x, y)$ is a penalty function for violating the budget constraint. [11] The consumer can only remember a fixed set of behaviors and their corresponding utility levels. At every time period the consumer reproduces and recombines his set of behaviors according to the adaptive model. If the model, in its present form, is directed towards only a single individual then it must be assumed that the consumer can calculate the utility of bundles which he has not actually experienced (this contradicts one of the arguments for the use of the adaptive model), furthermore, some method for choosing the actual bundle purchased must be employed (for example, the consumer could choose the best of the known bundles, randomly choose one of the remembered bundles either uniformly or weighted by the bundle's utility, etc.). These problems can be avoided by either using a more complex version of the genetic algorithm [12] or assuming that the model is being applied to a group of consumers, each of whom represents one behavior in the model, with identical utility functions who share all payoff information.

Simulations of the adaptive model were conducted under various price regimes. [13] Behaviors were assumed to be two 8-bit strings, which were evaluated as amounts of x and y to purchase ($x, y \in [0, 255]$). Consumers were assumed to be able to remember twenty behaviors at any time, or alternatively there were twenty consumers in the population. The payoff for each behavior was given by the following utility function: $U(x, y) = x^{0.75}y^{0.25} + 50 - \Gamma(x, y)$, where $\Gamma(x, y) = x^{0.75}y^{0.25} + 50\ \gamma^2/\gamma_{max}^2$, $\gamma = I - x - p_y y$, and γ_{max} is the maximum attainable value of γ. The interpretation of this utility function is straight forward: if consumers do not violate their budget constraint then they receive the utility specified by a simple Cobb-Douglas utility function; if the budget constraint is violated then the consumer is not allowed to purchase any goods and is penalized depending upon the extent of the violation. [14] The above model was simulated thirty times. Each simulation consisted of thirty periods. Behaviors were initially chosen at random. During the first 14 periods the price of y was set at \$1; from period 15 onward the price was changed to \$2. Income remained constant throughout at \$250.

The results of the simulation are presented in Figures 1a–1c. In each figure the average value in each period over all thirty simulations is shown bounded by a single standard deviation. In order to facilitate comparison with the optimizing model the optimal values are also presented in each graph. Figure 1a shows the average utility of all twenty behaviors during each time period. As expected, initial behaviors are quickly modified and the average utility approaches the maximum attainable utility. When the price changes

[11] An alternative formulation of the model would be to include the budget constraint directly, for example, allow behaviors to specify the relative percentages of income spent on the two goods. However, it was thought that the use of such a formulation would implicitly incorporate the optimization approach, and thus the more challenging formulation was used.

[12] Such a version of the algorithm does exist. In this version, at every time step only one behavior is chosen to undergo modification. However, in order to simplify the presentation of the original model this approach was not used.

[13] For all of the simulations conducted in this paper the following parameters were used: probability of cross-over = 0.75, probability of mutation = 0.01, and population size = 20. The cross-over and mutation parameters were within the range of values which DeJong (1975) established as providing good overall performance for the algorithm in a variety of environments. The population size was somewhat smaller than optimal, but it was thought that in order to maintain realism the smaller population size would be used with the expectation of genetic drift, and thus premature convergence towards near-optimal values.

[14] Note that since the payoffs are used by the model to form a probability distribution, the utility function is scaled such that it never attains negative values. The use of the genetic algorithm in constrained maximization problems is discussed in Goldberg (1983).

at period 15, utility plummets because many of the behaviors which were previously performing well now violate the budget constraint and thus incur large penalties. After the price change, behaviors are again modified and average utility approaches the optimal levels. Figures 1b and 1c detail the average demand paths for goods x and y respectively. As noted in Figure 1a, the demand for both goods rapidly adjusts towards optimal levels from the randomly chosen initial behaviors and the price shock in period 15. The average demand for y is particularly remarkable in its ability to achieve near-optimal levels in only a few periods of adjustment.

The above example illustrates the use and results of an adaptive model vis-a-vis a standard optimization model. The adaptive model is easily formulated, requiring only slight modifications of some of the elements of the optimizing model, and eliminating many others. The results indicate that while the adaptive model does eventually approach behaviors predicted by the optimization model, the adjustment process is not instantaneous. While the analysis conducted here is only preliminary, the results do appear promising, and it is likely that more sophisticated analyses will provide further insights.

5.2 An Application to Behavior Under Uncertainty

An interesting test of the model under conditions of uncertainty is a probability matching experiment. Simon (1959, pp. 260-1) suggested that the empirical results of this experiment directly challenge the optimization approach of economic theory. In the basic probability matching experiment, subjects are repeatedly asked to choose one of two payoff devices. The payoff devices are designed to pay a fixed prize to the subject if the subject chooses the paying device, where the paying device is determined with a fixed probability p and $1-p$ respectively. Empirically, subjects tend to randomize their choices of devices in a way which closely corresponds to the payoff probabilities.

The above scenario has many economic analogues. One example, explored by Rothschild (1974) and Schmalensee (1975), is that of a shopkeeper who can set one of two prices, and is rewarded with a sale according to some given probabilities. The above example can also be formulated as the repeated Battle of the Sexes problem from the theory of games. This game is essentially a matching game, providing a reward to both players if their decisions match. An economic example of such a game is a firm (political party) which is contemplating whether to use a liberal or conservative advertising strategy, where the success of the campaign will depend on whether the public has a liberal or conservative mood.

To model the above situation 12-bit structures were used to represent the probability of the individual choosing the first payoff device. The individual was assumed to be able to remember twenty such structures. The first payoff device had a probability of payoff equal to 0.75. Two experiments were run. In the first experiment each structure was evaluated once during each time step, while in the second experiment each structure was given five draws of the payoff mechanism during each iteration. [15] During each time step, all structures were given an initial payoff of two points and received an additional two points every time they correctly chose the selected device. The basic problem was simulated 30 times.

The results of these experiments are presented in Figures 2a and 2b. From Figure 2a it can be seen that the model's behavior is consistent with empirical experiments. After an initial period of adjustment, the selection probability attains a plateau which closely corresponds to the probability used in selecting the active payoff device. There is however a large amount of variance around the average path. This variance is understandable in light of the single trial of the structures per time step which makes the payoff function highly stochastic. To eliminate some of this effect, the second experiment was conducted. In this experiment each structure was given five trials at each time step. The results of the multiple trial experiment are presented in Figure 2b. As can be seen, not only did the variance diminish, but the probability of selecting the better performing structure increased by about 0.10. This result is not too surprising since the longer the number of trials the better the estimate of a structure's true value, and consequently one would expect that if the length of trials was extended even further structures corresponding to the optimal strategy (i.e. probability of selecting the higher payoff device equal to 1) would be developed.

The probability matching experiment is an interesting application of the model. It illustrates the use

[15] For each of these draws, one random number was drawn to determine which payoff device was selected, and another was drawn to determine which device each structure would have chosen. The same random number was used for all of the structures on a given draw, implying that the probability evaluation of each structure would have been made under the identical circumstances.

of the model under a highly stochastic environment. It also suggests a case where the implications of the adaptive model and optimization model diverge. Finally, this micro-level application of the model not only provides insights into a set of common economic problems, but it also allows some intuition into the underlying psychological basis of the overall model.

5.3 An Application to Market Structure

Adaptive models provide an interesting analytical technique for exploring the dynamics of market structure. The interaction of firms over time in any given market is a complex process. Although equilibrium models provide general notions about the directions of market systems and possible steady states, they do not easily account for the intricate dynamics. Adaptive models, however, can easily integrate dynamic problems, and therefore are well suited to the analysis of market structure.

The biologically-based model presented here provides a particularly good representation of market structure. Markets have very close parallels in biological ecosystems. Like organisms living in the same ecosystem, firms operating in the same market must compete with one another to survive. Firms which can compete successfully will flourish and grow in size; firms which can not compete will eventually go out of business. Although the model presented below is rather simple, it does provide some interesting insights.

This analysis will explore the market structure of a standard perfect competition model. The model assumes that a group of firms, producing homogeneous outputs, operate in the same market. The market price facing each firm is given by the inverse demand curve $P_t(Q_t) = a - bQ_t$, where P_t is the price prevailing at time t, $Q_t = \sum_i q_t^i$, q_t^i is the output of firm i at time t, and a,b are constants greater than 0. Each firm faces the same cost function $c(q_t^i)$. Under these conditions the profit for each firm i, π_t^i, is equal to $P_t q_t^i - c(q_t^i)$.

An adaptive model was implemented for the above assumptions. Twenty firms were assumed to operate in the industry which had an inverse demand curve of the following form: $P_t = 10000 - 0.125Q_t$. Each firm adapted its output level, described by a 12-bit binary number ($q_t^i \in [0, 4095]$), based on the payoff function $\mu_t^i = \pi_t^i + 2000000$ for $\pi_t^i \geq 0$, and $\mu_t^i = 2000000 + 0.10\pi_t^i$ for $\pi_t^i < 0$. [16] Thus, the higher the profit the better the firm's performance. Note that to maintain positive payoffs, a requirement of the model's payoff function, a firm's losses were scaled such that higher losses resulted in poorer performance. [17] Initial output levels for the firms were chosen at random. The above model was simulated thirty times under varying cost conditions, and total industry output and the Herfindahl-Hirschman (H-H) [18] index, a measure of industry concentration, were collected for each of the thirty periods the model was run.

Two types of cost conditions were imposed on the model. The model was first simulated under conditions of constant marginal costs, more specifically $c(q_t^i) = 5000q_t^i$. In the second simulation conditions of increasing marginal cost were assumed: $c(q_t^i) = 1.25(q_t^i)^2$. These cost functions were designed so as to give the same perfect competition equilibrium at $q_t^i = 2000 \; \forall \; i$. A priori, one would expect that the market dynamics would be different under the two cost conditions. In the case of constant marginal costs, if the market price exceeds the \$5000 marginal cost, any firm can increase its profits by producing more output. Unfortunately, if all firms attempt to do this the market price may fall below \$5000 implying losses for all of the firms in the industry. In the case of increasing marginal costs the incentive for a firm to increase its output is reduced, since even without a market price decrease increases above some limit will reduce profits (in this case above the point where $q_t^i > P_t/2.5$). Thus, one might expect more stability in the second case.

The simulations confirm the a priori expectations. Figures 3a and 3c present the average of the total industry output for the two cases. In the case of constant marginal costs (Figure 3a) output levels are always above the output level predicted under perfect competition ($2000 \times 20 = 40000$). This reflects the large incentive for firms to increase their output levels at near-equilibrium price levels. In the case of increasing marginal costs, the industry output level closely oscillates around the expected equilibrium output.

[16] The model was somewhat sensitive to the form of the payoff function. If the payoff function was formulated without reference to the initial conditions, it could easily occur that large negative profits would be earned by all firms. This would result in inadequate differentiation between the production plans, and therefore nullify the effectiveness of reproduction by performance and the cross-over operator.

[17] This is similar to the use of penalty functions discussed in the application to consumer demand.

[18] This index is defined as H-H $= \sum_i s_i^2$, where s_i is the market share of firm i.

More insight into the actual dynamics can be obtained from the H-H index levels recorded in Figures 3b and 3d. Under constant marginal costs market conditions the H-H index remains fairly constant at about 0.066, while under increasing marginal costs the index rapidly falls from its initial value of 0.067 to a relatively stable 0.054, closely approaching the perfectly distributed index value of 0.050. [19]

The model of market structure implemented above is only one example of the potential use of an adaptive approach. In the simplified case above, firms were only allowed to adjusted their level of output to prevailing market conditions. However, one of the strengths of the adaptive model formulated here is its ability to easily incorporate multiple criteria in the formation of new decisions—this power is especially evident in the cross-over operator. Thus, a market analysis including such factors as advertising decisions, product quality choices, etc., would certainly be feasible.

Adaptive models are an appropriate choice for the analysis of dynamic systems. As in the case of market structure above, the model can easily be formulated to encompass a complex dynamic system. Once formulated, useful hypothesis about the behavior of such systems can be generated. Given the importance of dynamic economic systems to both theoretical and policy issues, such analyses would prove valuable.

5.4 An Application to Technological Change

Studies of technological innovation have indicated that its economic impact may be substantial (Solow, 1957). Unfortunately, optimization models of technological change have not been able to reconcile the existing empirical knowledge (for a summary see Nelson and Winter, 1982, pp. 202-4). The inherent dynamic nature of innovation along with the necessity to explore previously unknown processes, present a formidable task to maximization models. Adaptive models, with their explicit dynamic nature and low information requirements, are a natural choice for the study of innovation.

The genetic model allows an interesting analysis of technological change. The model admits the possibility of technological innovation (via the mutation operator), and also allows firms to imitate successful innovations to varying degrees (through the reproduction and cross-over mechanisms). This approach is consistent with the empirical reality of the innovative process. The model presented here assumes that the technological innovation is initially unknown to the firms and is only discovered stochastically in later time periods. The innovation is initially unknown due to either pure technological constraints, for example, the use of atomic power prior to 1940, or constraints imposed by tradition, for example, the organizational structure of the firm. [20] Industry demand is assumed to be infinitely elastic and inputs are assumed to be costless, thus, all firms attempt to maximize output. [21]

The model assumes that each firm i in the industry has a Cobb-Douglas production function of the following form: $q_i = A_i^{0.25} K_i^{0.5} L_i^{0.5}$, where A_i, K_i, and L_i are the respective amounts of the innovative, capital, and labor inputs used by firm i. The innovative input is a factor of production which has been set by all firms to the same value either by tradition or by previous technological constraints. [22] The industry is simulated for thirty periods. Initially, the twenty firms in the industry randomly choose production plans, described by three five-bit numbers (with $A, K, L \in [1, 32]$), constrained only by the fact that A was equal to 1 for all firms. After the 11th period, the innovative factor of production was no longer constrained, and firms were allowed to vary the amount of A. The payoff to each firm was simply the quantity it produced. Each simulation was conducted thirty times, and the results are presented in Figure 4.

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[19] For comparison purposes, if 19 of the firms in the industry had equal market shares then an index value of 0.066 would imply that the last firm had a market share of 17.3 percent whereas a value of 0.054 would imply a market share of 11.2 percent.

[20] An interesting implication of the problem of genetic drift (that is, the loss of initially poor performing yet ultimately important alleles to the entire population), is that an industry could get entrapped by tradition.

[21] The imposition of a more complex demand structure and input prices could easily be incorporated.

[22] A more complex functional form, where the marginal product of A_i was negative for small levels of A_i, was also tried. It was thought that this production function might provide a better justification for the lack of industry diversity in the use of A: every time a firm had tried to increase its use of A it had lower output. This example may be consistent with some forms of technological change, for example, mass production where low levels of utilization are not productive, and it is only when a certain threshold level is passed that returns increase. However, the results of this more complex functional form were not qualitatively different than the results of the simpler form, and therefore were not presented.

Figure 4 conforms with a priori expectations. Initially, firms steadily increase output levels starting from their randomly chosen production plans. By the 11th iteration the average industry output is close to the maximum achievable level. After the 11th iteration innovation is allowed in the industry. The way innovation occurs is through the mutation operator randomly changing one of the bits describing the amount of A to use in production (the probability of this happening during any given period is 0.634 ($= 1 - .99^{100}$)). Once innovation occurs, other firms are likely to incorporate the use of the innovative output with their current production plans. This results in relatively rapid output growth in the industry. The period of rapid growth noticeable slows after about 10 periods and a slower transition towards the optimal output level ensues. [23]

In order to separate the process of innovation from that of imitation the initial model was simulated under two variations. The first variation, designed to capture the innovative component, allowed only mutation to operate in the model (that is, each firm was always given one offspring and no cross-over was allowed). The output path of this simulation is given in Figure 4. It is apparent that with only innovation operating in the environment almost no increase in productivity occurs. The second variation of the model allowed mutation and reproduction according to performance, but no cross-over. Figure 4 indicates that this variant had an output path which was almost identical to that of the initial simulation, with only a slight degradation of output levels in latter periods. This output path indicates that the importance of imitation was not in recombining any innovation with existing goods, but only in imitating more successful innovators. This result is consistent with the form of the production function used in these experiments, since the innovative input provides a monotonic increase in output regardless of the configurations of the other inputs.

The genetic model is a good choice for the study of technological change. The dynamic market context in which most innovation takes place makes the adaptive approach appealing. Moreover, the strong innovative and imitative components of the genetic model closely match the empirical reality of technological change. As the example above illustrates, by varying the parameter values of the model the relative importance of such effects can be effectively examined, and insight into a problem which was previously inaccessible can be gained.

5.5 An Application to Economic Demography

The transition of societies from conditions of high fertility and mortality to conditions of low fertility and mortality has been an important topic of much social science research. While initial theories apparently accounted for the empirical reality, recent evidence, most notably from the European Fertility Study conducted by the Office of Population Research at Princeton, has raised a number of questions about the actual mechanisms involved. This research indicates that an adaptive approach may be an effective way to model the phenomenon of the demographic transition.

Studies of Europe's demographic transition have revealed many interesting results (for a summary see van de Walle and Knodel, 1980). Before the transition most populations experienced a natural fertility regime, whereby couples did not consciously limit their family size. The onset of the transition is not consistently explained by the level of industrialization, urbanization, education, or mortality changes. However, there is strong evidence that the innovation of methods of fertility limitation at the start of the transition may have played a key role. Once the transition started a rapid and monotonic change towards substantially lower levels of fertility and mortality occurred, generally following regional linguistic and cultural clusters.

A macro-level adaptive model captures many of the components inherent in the demographic transition. The strong linguistic and cultural components of the transition, along with the apparent impact of migrants from areas experiencing the transition on pre-transitional areas, implies that the flow and diffusion of information and ideas was important. The evidence of the innovation of family limitation technology also supports an adaptive hypothesis.

A relatively simple adaptive model captures some of the major components of the demographic transition. Suppose that couples are only concerned about the number of surviving children, and attempt to achieve an "ideal" family size. This can be operationalized by the following utility function, $U(B_s) = A - (|B_s - B^*|)^{0.5}$, where A is a constant, B_s is the number of surviving children, and B^* the ideal family

[23] It is likely that the reason why more rapid convergence towards the optimum doesn't continue is the initial lack of "genetic" diversity in the behavior describing the amount of the innovative input to use.

size. The number of surviving children, $B_s = Bcp$, where B is the number of births without any fertility limitation, c is a measure of contraceptive effectiveness ($c \in [0,1]$), and p is the probability of surviving from birth to adulthood.

Two simulations were conducted. In both of the simulations individuals were allowed to adapt 10-bit structures which represented the effectiveness of fertility control. Initially, all of the structures had a value of 1 (that is, natural fertility) and innovation was allowed to take place. In the first simulation the following parameters were used: $A = 4$, $B = 11$, $B^* = 2$, and $p = 0.5$. The second simulation used identical parameter values with the exception of the survival parameter which was allowed to increase by 0.05 each period until it achieved a value of 0.8. This was intended to simulate the concurrent mortality decline experienced by some of the European countries and most developing countries today. Each simulation was carried out thirty times.

The results of the simulations are shown in Figures 5a–5d. Figure 5a (5c) shows the transition path of the average fertility control parameter for the first (second) simulation, and Figure 5b (5d) graphs the corresponding average births and surviving children. The figures indicate that once some innovation is introduced into the population (this occurs with probability $0.866 = 1 - (1 - 0.01)^{200}$) a rapid transition, within about 12 iterations, can occur. A comparison of Figure 5a with 5b indicates that with a declining mortality rate, the transition occurs at a much more rapid pace.

The inherent dynamic and innovative components of the demographic transition make an adaptive modelling approach promising. Although the above analysis is only preliminary, it does indicate that likely transition paths can be generated and compared. Through the use of such an approach it is likely that a better understanding of the important empirical facts surrounding the demographic transition can be obtained.

6. Conclusion

In all of the above examples the adaptive model appears to be a valuable approach to modelling economic and social phenomenon. In general, the results appear extremely promising. The model has proven to be very flexible, and was easily formulated for a variety of diverse applications and environments. In each of the applications the model performed admirably. In those applications where more orthodox solutions existed, the outcomes of the two models often corresponded. When the predictions of the two models differed, the adaptive model appeared to be more consistent with the existing empirical reality. The applications indicated that the transition paths of the model are relatively consistent, and can be usefully analyzed. [24] By changing the parameter values of the model, qualitatively different transition paths were generated, and therefore a form of comparative "statics" is available. The model did not appear to be sensitive to minor variations in the parameter values. However, unlike optimization models, it is sensitive to the form of the payoff function. [25] Although all of the examples used relatively few behavioral parameters, other empirical studies of the model indicate that its performance with multiply parameters is not substantially diminished, and therefore this approach should provide an effective technique for modelling much more complex forms of behavior.

The theoretical properties of the model along with the empirical results from above, provide a great deal of insight into the behavior of the model. The dynamic nature of the model is explicit and of paramount importance to its implications. Some general results are: (1) that given *enough time* and the proper conditions agent's behaviors will tend towards optimal patterns, (2) that adjustment to new environmental conditions is initially very rapid, but quickly slows as the behaviors approach optimal levels, and (3) that various parameter values may generate testable hypotheses (for example, smaller population sizes may be expected to rapidly adapt to near-optimal behaviors, large mutation rates prevent premature convergence at the expense of poor long term performance, low cross-over rates may depreciate performance, and various parameter settings

[24] The statistical techniques used in all of the applications are relatively primitive. More appropriate techniques, such as multiple regression methods designed for panel data, could be employed in more sophisticated analyses.

[25] Since the model uses the payoff function to form a probability distribution over the behaviors, the form of the function is important. Payoff functions have to always be positively valued, and transformations which affected the ratio of the function's value relative to its mean may lead to different outcomes. These restrictions may hamper the use of the model in some situations.

may influence the variance of the expected path). These results indicate that the model can be usefully employed in further economic analyses.

By relying on specific biological concepts, an interesting model of adaptive economic behavior has been developed and analyzed. The reliance on specific biological processes has reduced the ambiguity which has confounded previous biological-based models, and has provided sufficient structure to explicitly explore the model. The basic assumptions of the model were examined, and they appear to accord well with current economic and psychological theories of behavior. The model was readily applied to a variety of phenomenon and interesting insights and implications were obtained. Furthermore, based on the pioneering work of Holland et al., the theoretical properties of the model were derived. These properties indicate that the basic model has a strong optimization component, [26] which suggests that the optimization and adaptive approaches are not mutually exclusive paradigms. The relatively simple foundations of biologically-based models along with their wide applicability and interesting implications, naturally make them a good candidate for further analysis.

[26] In fact, the cited empirical studies indicate that the model outperforms current optimization techniques, which one assumes maximizing agents will be employing.

APPENDIX
THE GENETIC ALGORITHM

The genetic algorithm used in this analysis has the following form:

1. Initialize $A(t)$ by choosing M structures at random.
2. Set $t = 0$.
3. Calculate the performances of each structure i in $A(t)$:
   a. Let $V[i] = \mu_t(A_i(t))$.
   b. Let $\hat{\mu}_t = \sum_{i \in A(t)} V[i]/M$.
4. Reproduce structures in $A(t)$. For each $i \in A(t)$:
   a. Allow TRUNC$(V[i]/\hat{\mu}_t)$ offspring.
   b. Allow an additional offspring with probability $(V[i]/\hat{\mu}_t)$ − TRUNC$(V[i]/\hat{\mu}_t)$.
   c. Adjust the population if the number of offspring is not equal to M.
   c1. If the number of offspring $< M$ then allow the best performing structure in the new population to produce additional offspring.
   c2. If the number of offspring $> M$ then eliminate the poorest performing structure(s).
   d. Randomly mix the reproduced structures.
5. Cross-over the newly reproduced structures. For each pair in the new population do the following:
   a. With probability P_c:
   a1. Randomly select a cross-over point $x \in [0, l-1]$.
   a2. Form two new structures by crossing over the two parents immediately after the xth allele.
6. Mutate structures. For each locus of each structure do the following:
   a. With probability P_m change the allele value at the locus to its alternative.
7. Let $t = t + 1$.
8. If $t < 30$ goto step 3.
9. End.

All simulations were conducted on an IBM PC1 or Zenith 150 equipped with an Intel 8087 math co-processor. The program was written in Pascal and was compiled using Turbo Pascal–87 version 3.0. Psuedo source code is available upon request.

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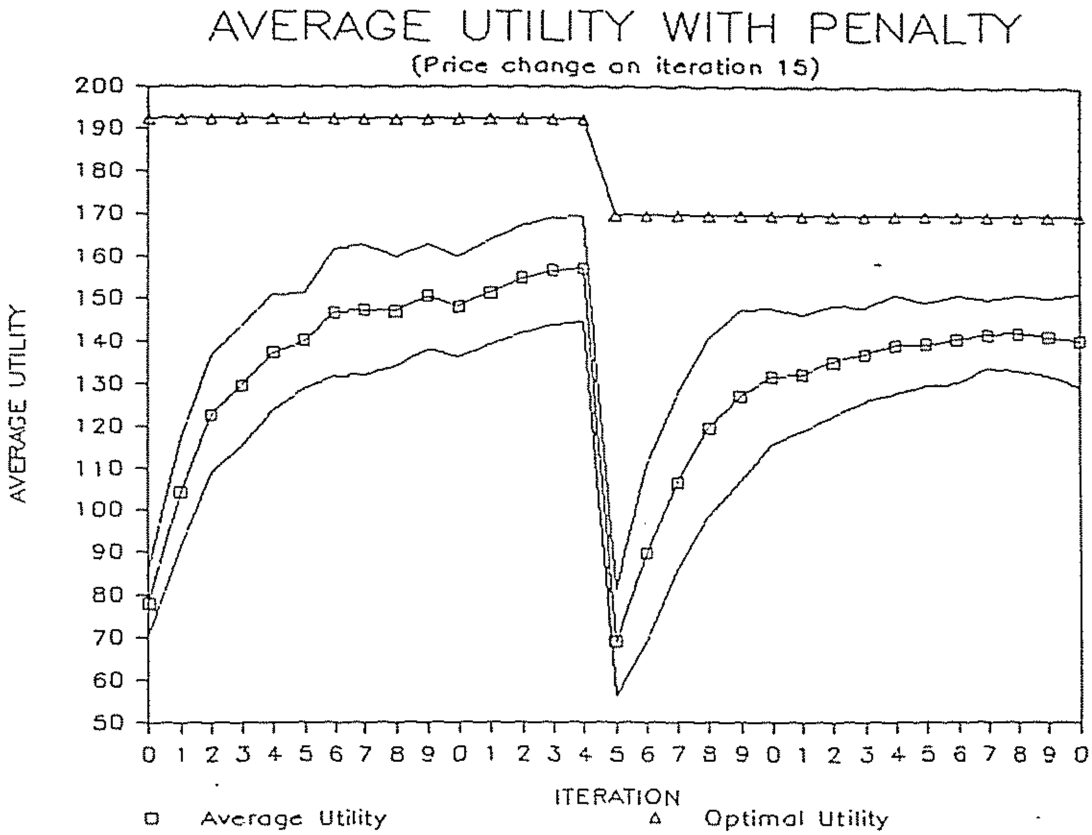

Figure 1a

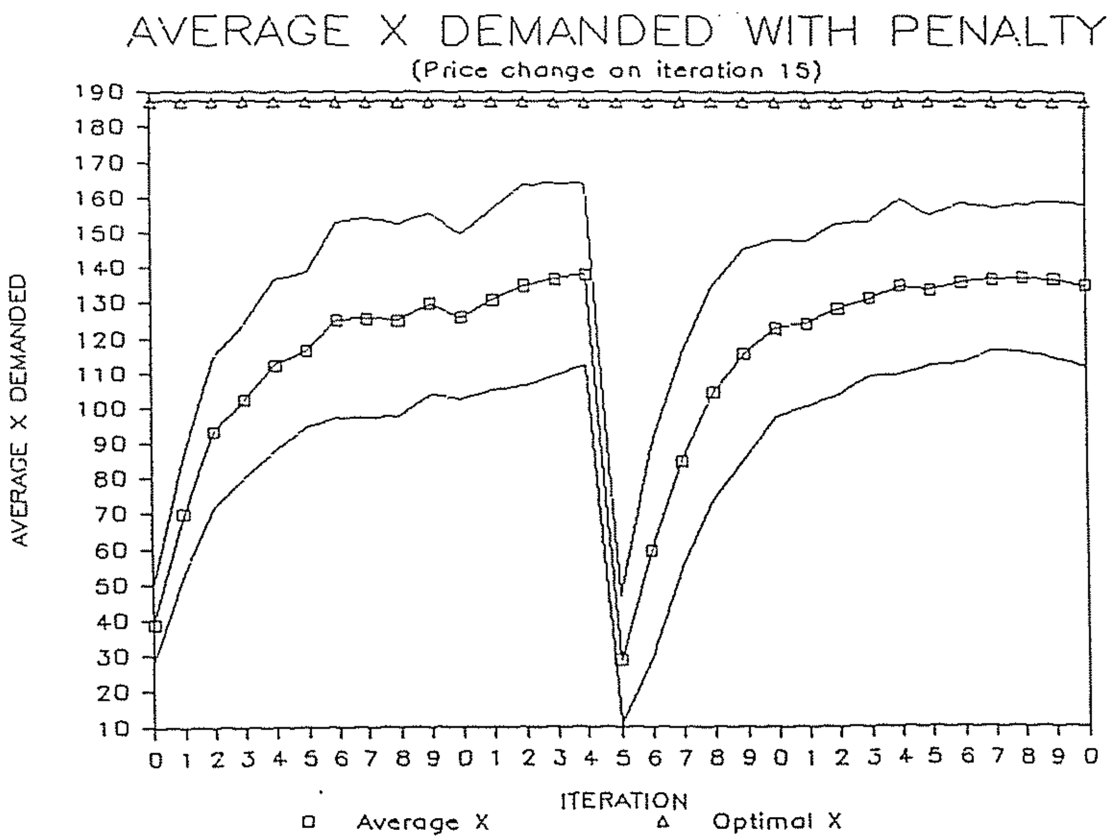

Figure 1b

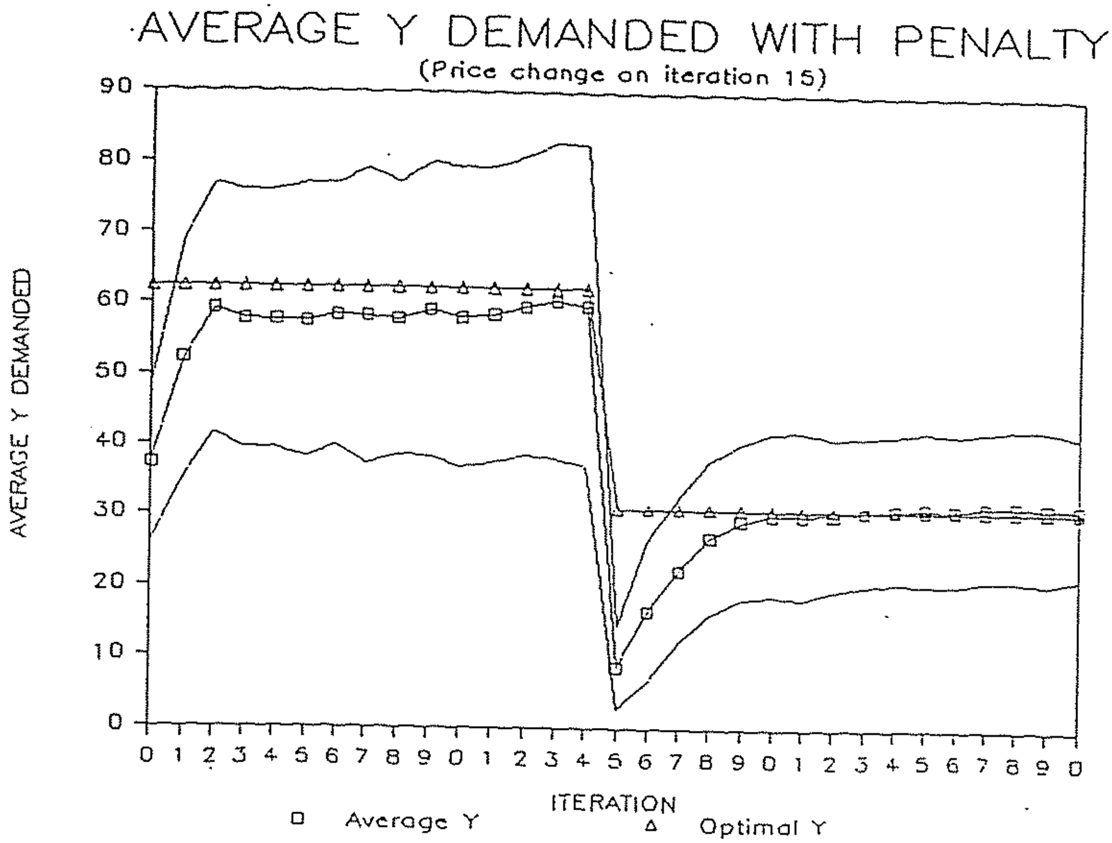
AVERAGE Y DEMANDED WITH PENALTY
(Price change on iteration 15)
AVERAGE Y DEMANDED
90
80
70
60
50
40
30
20
10
0
0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0
ITERATION
Average Y
Optimal Y

Figure 1c

PROBABILITY MATCHING EXPERIMENT

(Probability payoff right = 0.75)

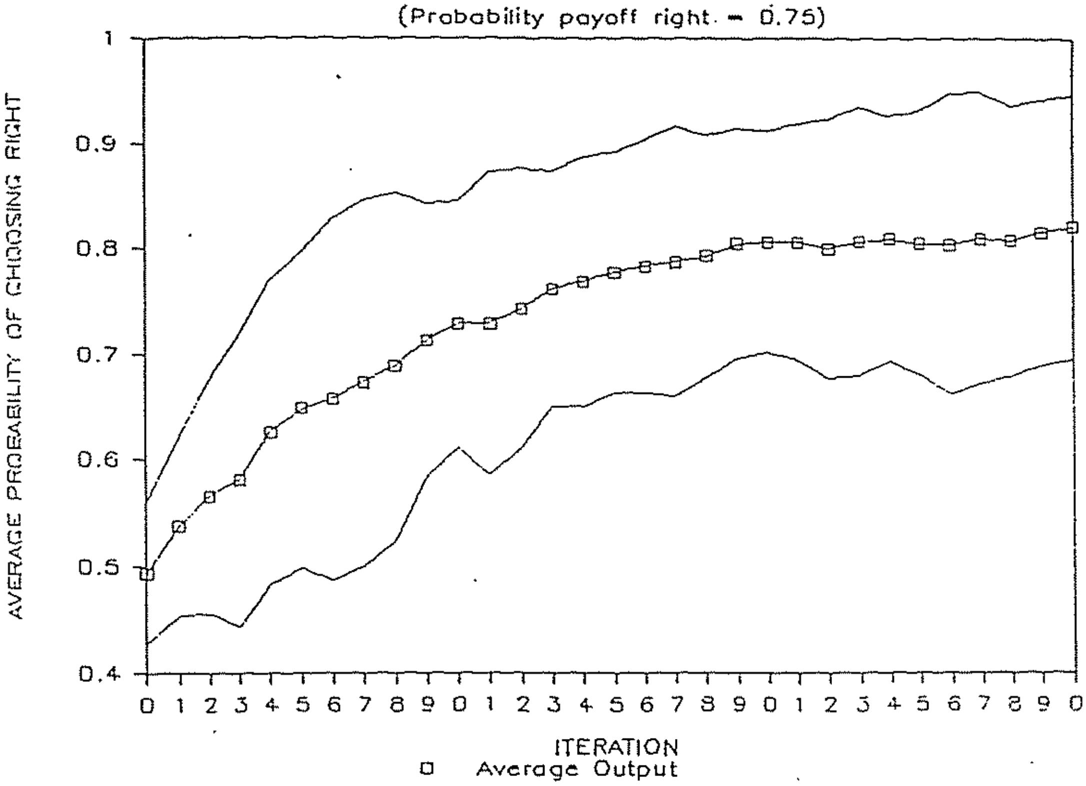

Figure 2a

PROBABILITY MATCHING EXPERIMENT

(Probability payoff right = 0.75)

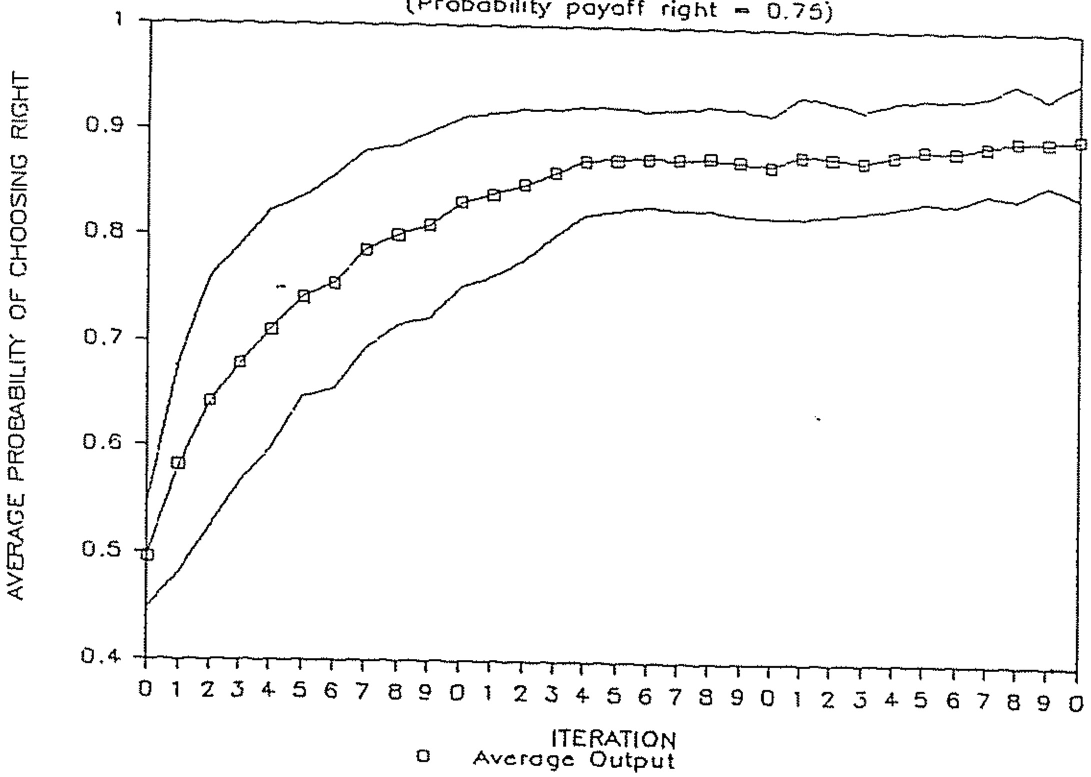

Figure 2b

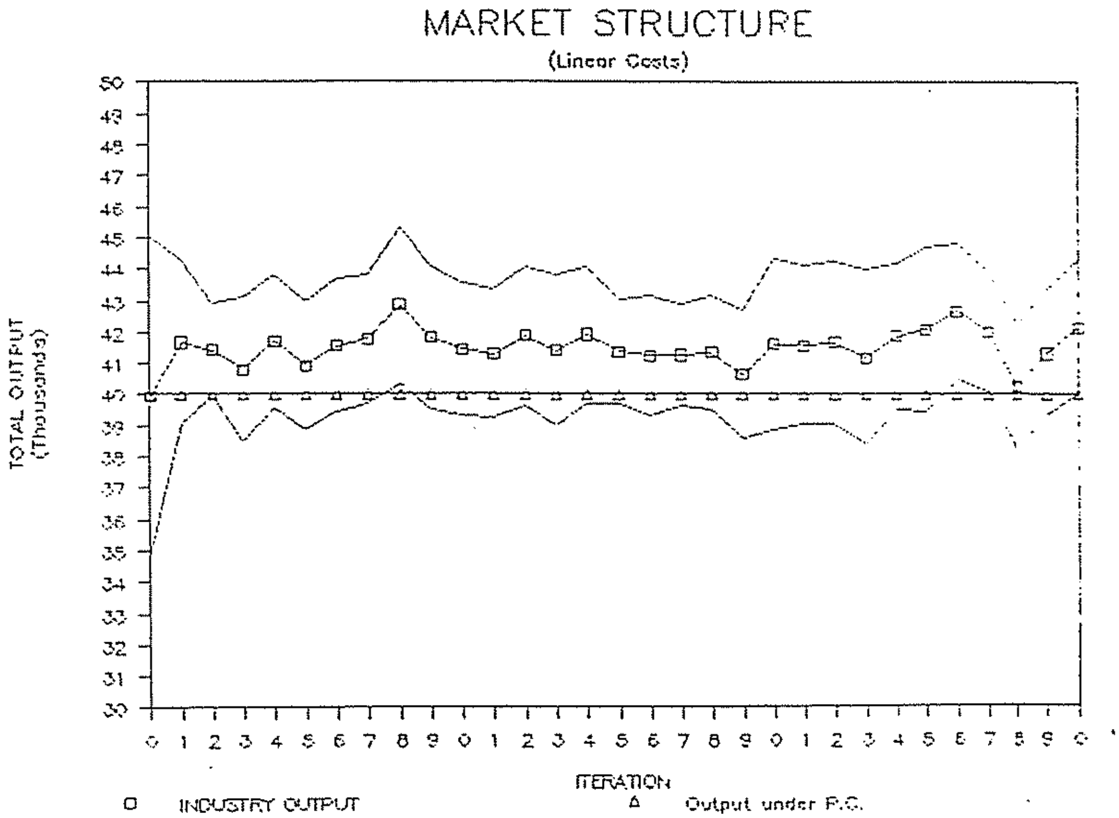

Figure 3a

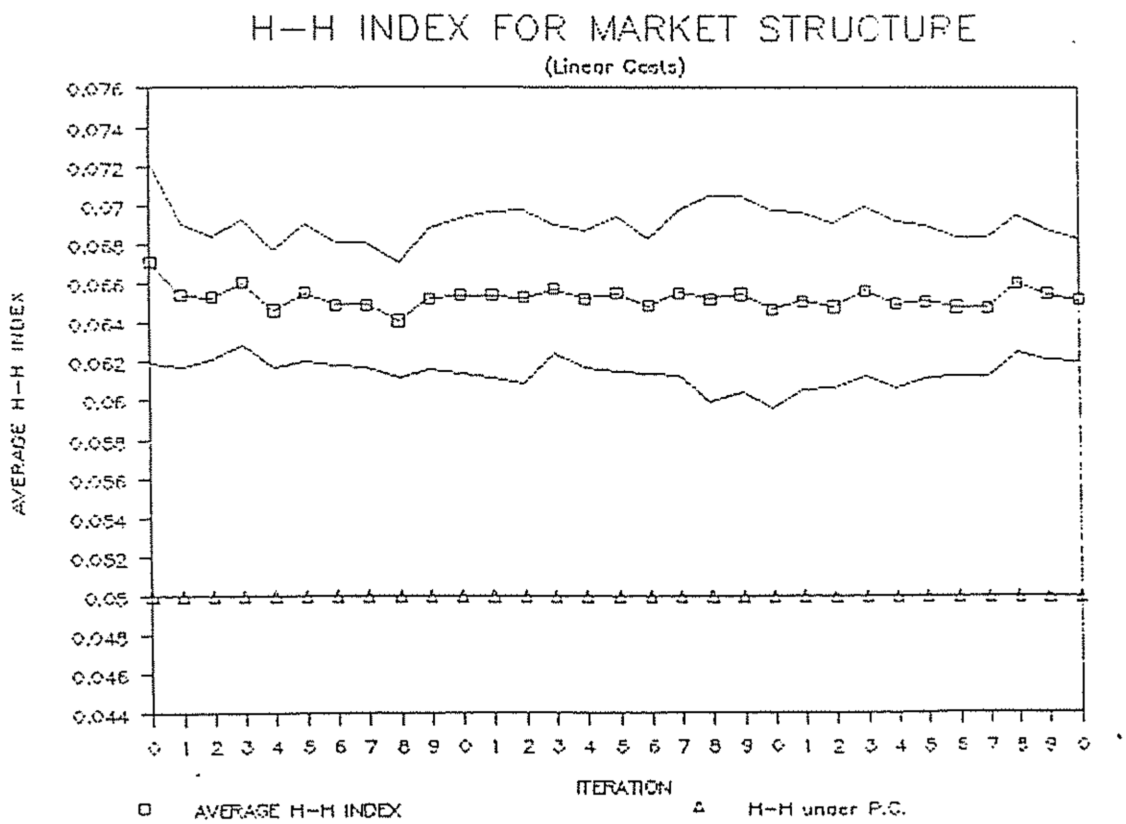

Figure 3b

MARKET STRUCTURE
(Quadratic Costs)

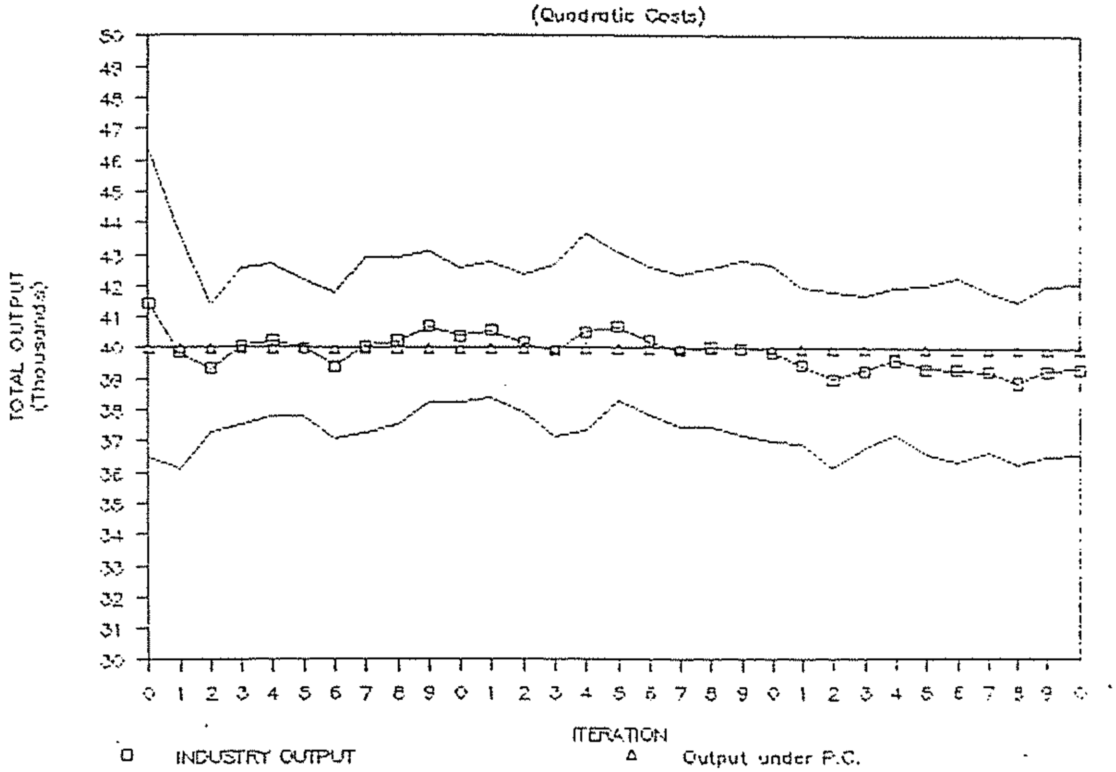

Figure 3c

H—H INDEX FOR MARKET STRUCTURE
(Quadratic Costs)

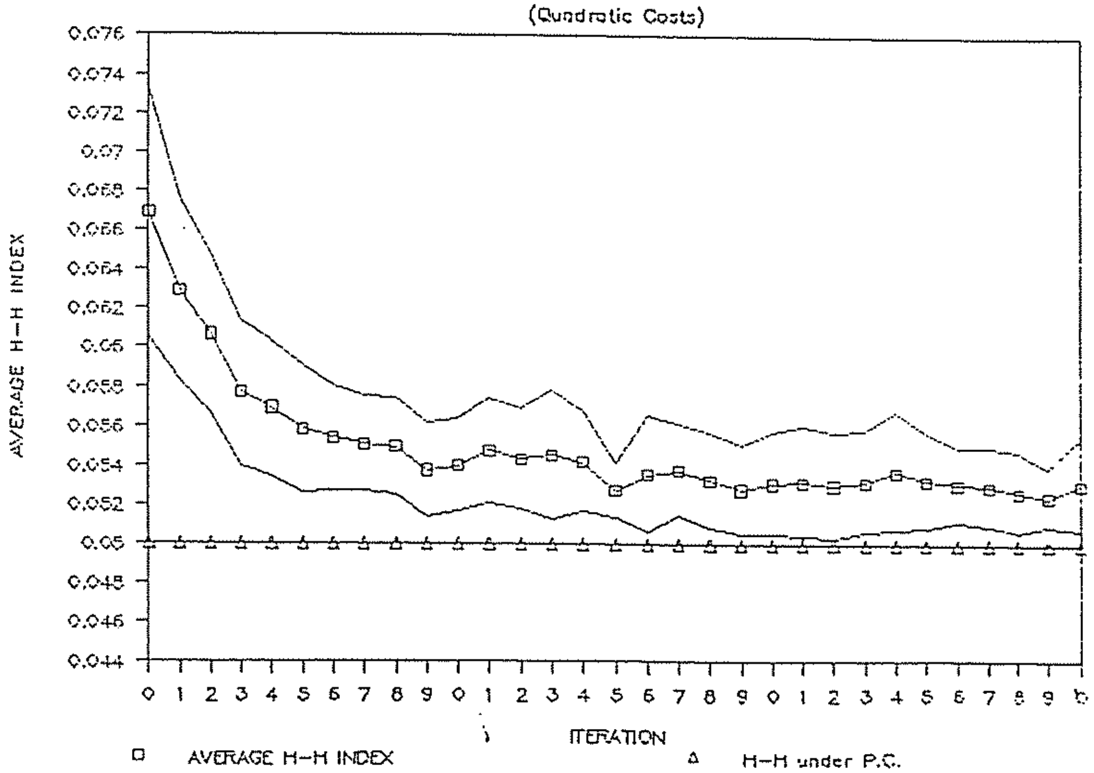

Figure 3d

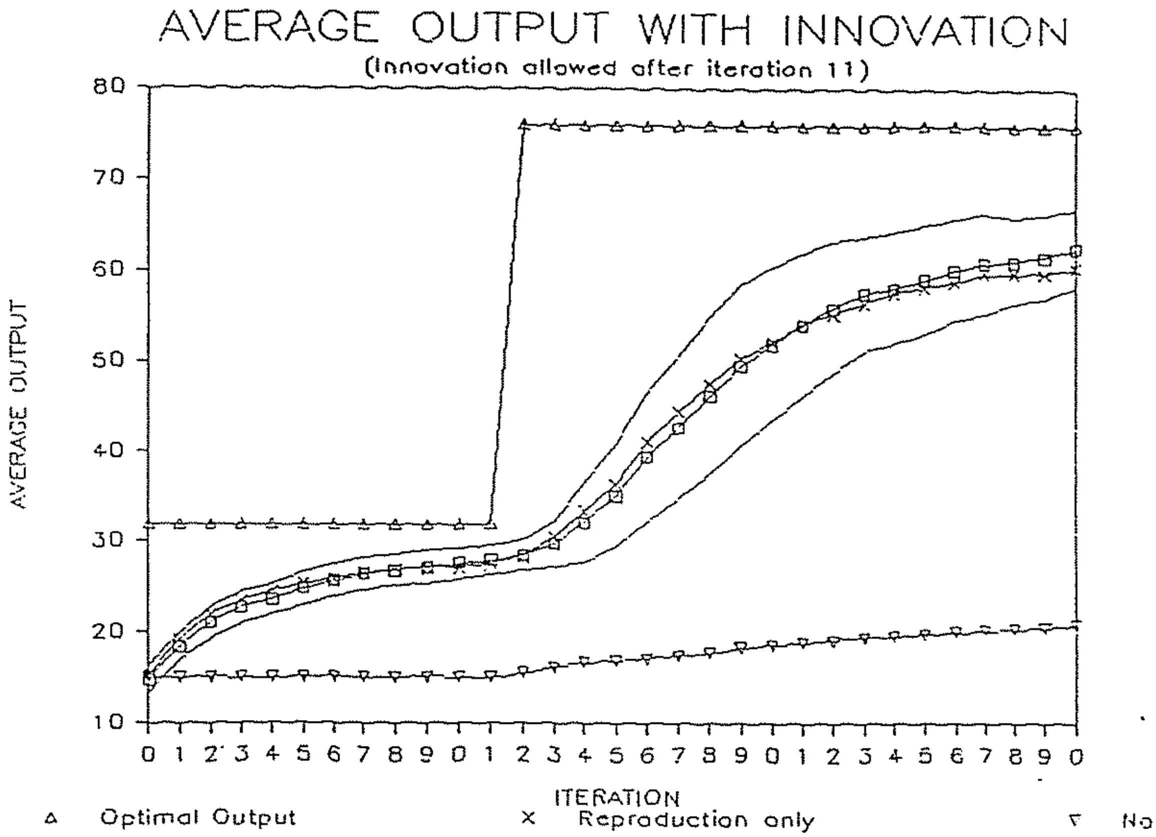
AVERAGE OUTPUT WITH INNOVATION
(Innovation allowed after iteration 11)
AVERAGE OUTPUT
ITERATION
Optimal Output
Reproduction only
No

Figure 4

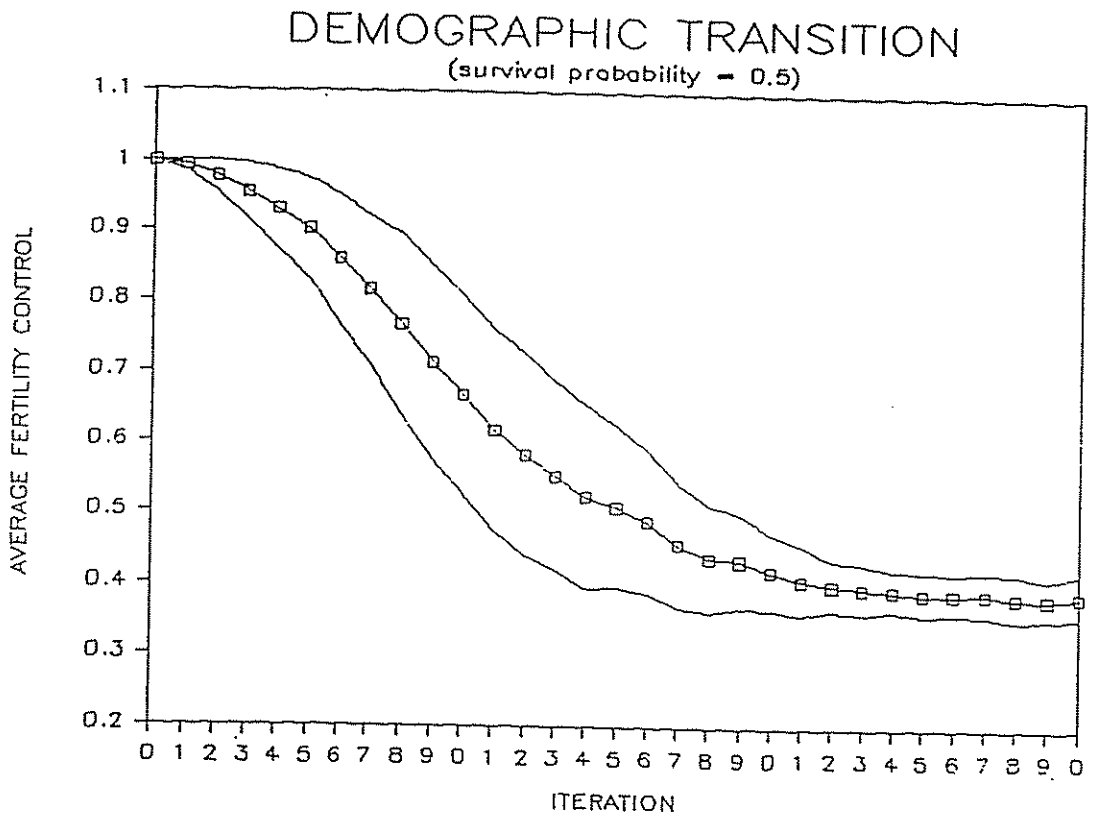

Figure 6a

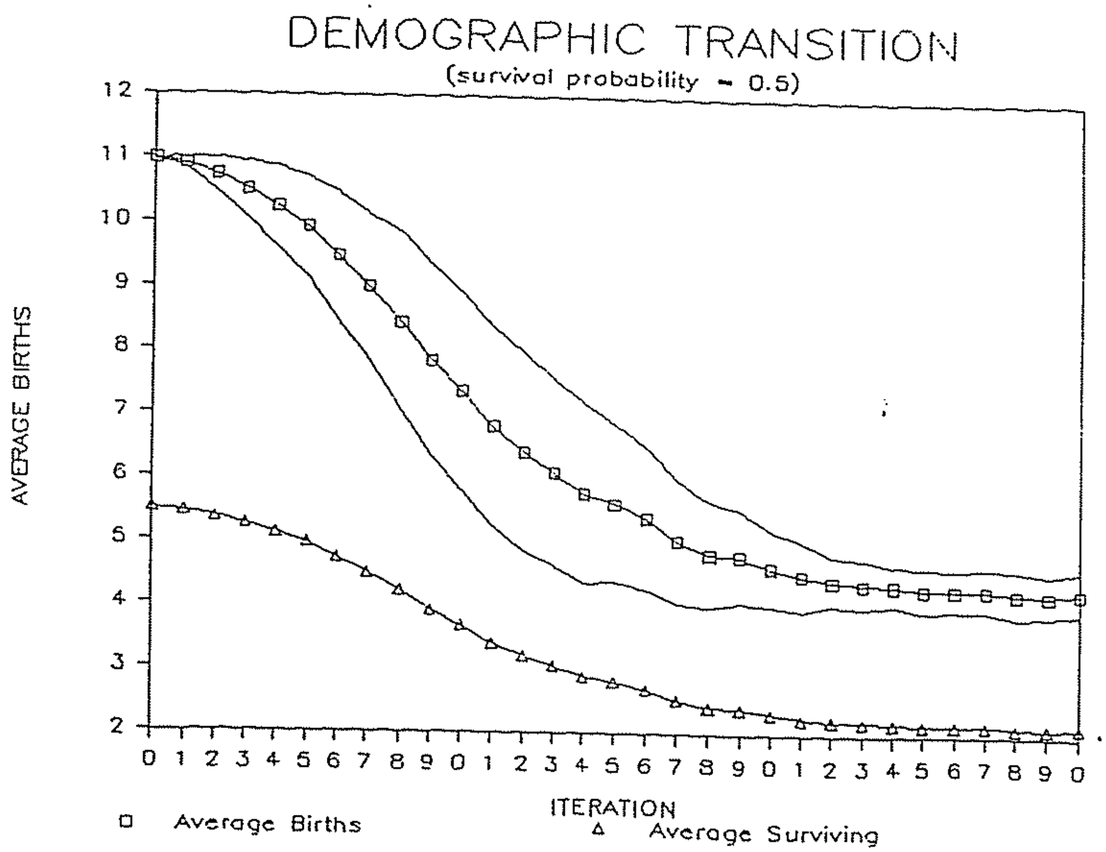

Figure 6b

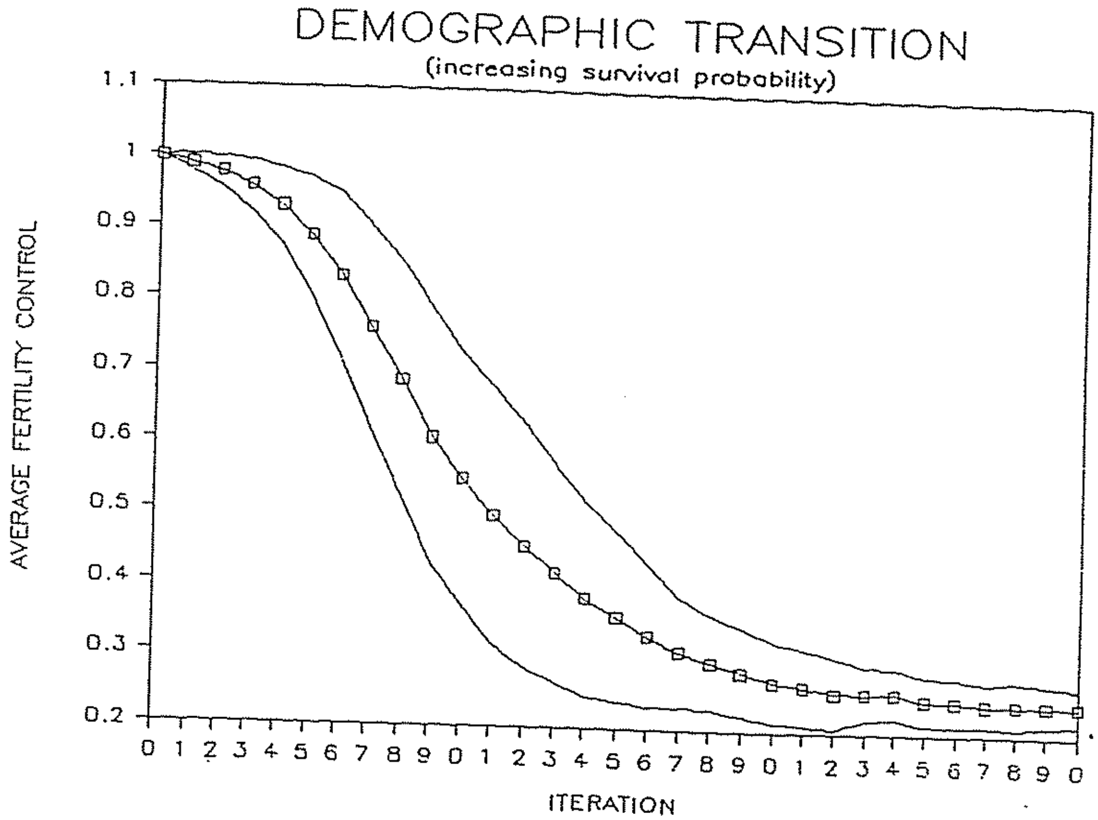

Figure 6c

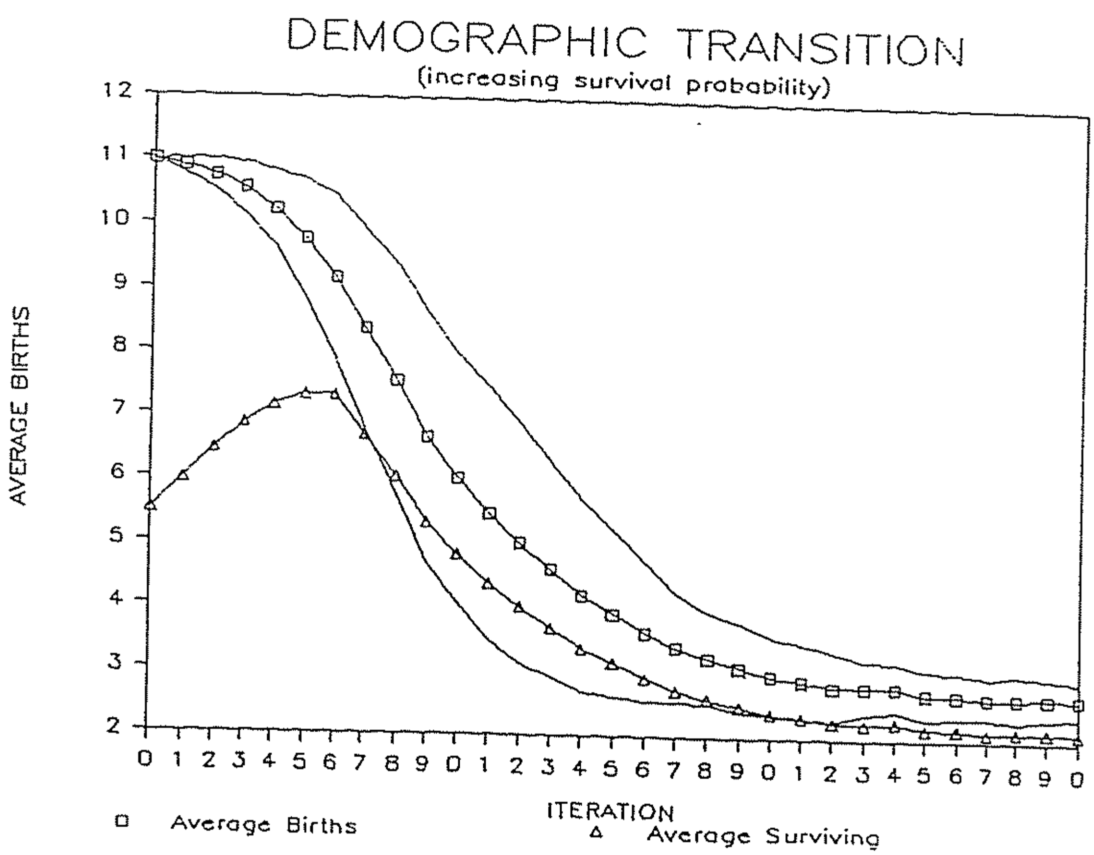

Figure 6d