

## ADAPTIVE PARTIES IN SPATIAL ELECTIONS

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**W**e develop a model of two-party spatial elections that departs from the standard model in three respects: parties' information about voters' preferences is limited to polls; parties can be either office-seeking or ideological; and parties are not perfect optimizers, that is, they are modelled as boundedly rational adaptive actors. We employ computer search algorithms to model the adaptive behavior of parties and show that three distinct search algorithms lead to similar results. Our findings suggest that convergence in spatial voting models is robust to variations in the intelligence of parties. We also find that an adaptive party in a complex issue space may not be able to defeat a well-positioned incumbent.

**S**ince Anthony Downs's *Economic Theory of Democracy* (1957), a spatial theory of elections has occupied a prominent theoretical status within political science. Modelers use the intuitive notion of ideological distance to develop explanations for observable electoral trends. The most famous of these trends is the Downsian idea that in a two-party system, given certain assumptions, parties converge toward a median position on the continuum of possible voter positions. Although simple spatial models have produced this result, extensions to the models have questioned the robustness of this prediction. Following the voting paradox and the results of Plott (1967) and McKelvey (1976), some scholars have speculated that chaotic results are possible and, in some cases, likely. In two or more dimensions, the top cycle set consists of the entire space of issue positions.<sup>1</sup> Bates summarizes the more general result: "The principle lesson is that, in general, one cannot expect an equilibrium to exist; and, because any outcome can be defeated, political decisions represent arbitrary outcomes" (1990, 45). Whereas some scholars concede the predicted instability in multidimensional voting models (Riker 1982), others see the Downsian model's stability and convergence as more empirically accurate and (perhaps) more normatively desirable.

Some theorists, inclined to believe that electoral chaos is extremely unlikely, have incorporated various complexities to explain stable, often centrist, outcomes. Coughlin (1990a) divides these models into four general categories: (1) models that allow for mixed strategies by parties, (2) models that track dynamic trajectories of party locations, (3) models that search for uncovered or undominated sets, and (4) models that include candidate uncertainty over voters' behavior (probabilistic voting models). He writes, "It is hard to resist the alternative inference that the primary contribution of recent work on the majority rule relation is as a grand 'reductio ad absurdum' that tells us to go back to the basic model that has been used to see how it should be modified

in order for theory and empirical observations to match up" (p. 164).

Two-party electoral outcomes appear more stable than the chaotic results predict, so the task of seeking alternative assumptions to get more realistic outcomes has a sound scientific basis. Nevertheless, both original spatial models and many contemporary revisions rely on unrealistic assumptions to produce equilibria. They often assume fully informed and optimizing voters and parties (Davis, Hinich, and Ordeshook 1970). Spatial modeling seems wedded to the defining assumptions of rational choice, namely, that people and organizations are self-interested, have complete information, and can locate optimal strategies regardless of complexity. Chaos results, for example, rely on unrealistic assumptions about the abilities of parties to locate winning platforms. Probabilistic voting models assume that parties position themselves optimally given complete knowledge of the probabilities of voters' actions (Coughlin 1990b). We find implausible even the restricted assumptions that parties have sufficient information and analytical abilities to locate optimally. Perhaps different assumptions about the abilities of parties or voters lead to stable outcomes.

We incorporate boundedly rational parties in a model of two-party competition. Instead of modeling parties as full-information global maximizers, we model them as incompletely informed and adaptive. They move incrementally toward better regions of the space through the use of search algorithms. Our model involves a dynamic interaction between parties who make decisions in an evolving environment. Much like Kramer (1977), we are interested in the trajectory of party platforms over a sequence of elections. Using our methodology, we are able to explore a variety of questions: Do imposed informational and computational constraints lead to arbitrary outcomes, with winning party platforms scattered throughout the space? Do boundedly rational parties converge toward centrist platforms? Do parties always defeat incumbents, as many models suggest? Does altering the preferences of parties from vote

maximizing to winning with ideals affect the behavior of parties? Our use of adaptive agents allows us to address these questions without wholly abandoning formal modeling.

We shall discuss the advantages of using artificial adaptive agents in the social sciences and present our basic model and a description of the two types of party preferences that we consider. We then present the three search procedures used by our parties and describe our results. We conclude with a discussion of the role of information in elections and possible directions for future research.

## THE USE OF ARTIFICIAL ADAPTIVE AGENTS

Holland and Miller advocate a class of models using artificial adaptive agents (AAA), a technique in which "the unfolding behavior of the models can be observed step-by-step" (1991, 366). Based on computer languages, these models are flexible enough to capture a variety of situations yet they maintain precision and logical consistency.<sup>2</sup> Computational models allow the exploration of systems of well-defined agents in a perfectly controlled environment that can be easily and rapidly replicated under many conditions. Moreover, any conceivable analysis is feasible since the state of the system is fully recoverable. This allows the researcher to generate, develop, and test theoretical hypotheses quickly. Although optimization is the key benchmark in much social science theory, learning, adaptation, and bounded rationality are recognized as important processes. Through the use of AAA models, questions about the relationship between optimization and adaptation can be explicitly explored. We adopt the AAA technique because of these advantages and because of our belief that political parties are better characterized as partially informed, finite-memory information processors who rely on rules of thumb than as completely informed rational agents.

Underlying our methodology is the notion that there exist important classes of behavior that can be captured in models too complex for traditional mathematical analysis. Absence of equilibria in a model (or equilibria that human agents could locate only by chance) does not necessarily imply a lack of predictability. AAA allow us to search previously inaccessible models for patterns of generic behavior. Our choice of technique has costs. Restriction to any one type of algorithm can yield ad hoc results, and predictions are often less precise than those of rational actor models. To control for these possibilities, we compare three distinct types of AAA, and arrive at strikingly similar results for all three. The use of multiple algorithms to discover equivalence classes of adaptive behavior distinguishes our model from others and strengthens our results.

There are many precedents for the use of AAA and similar techniques in the social and behavioral sci-

ences. For example, economists have applied AAA to evolve efficient bilateral trading rules (Marimon, McGrattan, and Sargent 1990) and to explore learning in varying economic environments (Arifovic 1989; Miller 1986). In political science, Cohen (1984) has used computer models to explore competing theories of organizational decision making, and Whicker and Strickland (1990) have used computer simulations to test the importance of opinion distribution and decision rules on the constitutional amendment process. Axelrod (1986) has simulated the evolution of norms and has also used a genetic algorithm to develop new strategies that were superior to Tit for Tat in his 1984 computer tournaments (Axelrod 1987). One of the present authors, Miller (1987), has analyzed the co-evolution of strategies and the emergence of cooperation in noisy repeated prisoner's dilemma games using adaptive algorithms.

Thus, we agree with Coleman (1989) that simulation models with humans, computers, or both can be useful in the construction of social theory. We shall use computer models to trace the emergent behavior of boundedly rational parties in the context of spatial voting models. Since our parties are limited in both the information they possess and in how they process the information, there is no guarantee that in finite time they will find optimal locations in the policy space. We can, however, compare the general movements of the parties over time to the predictions of mathematical models of party competition. The model we put forth does not stretch the boundaries of our technique. Our present purposes—to address a central problem in positive political theory and show the strength of our approach—are best served by a simple model.

## THE BASIC MODEL

Our model incorporates many of the assumptions of spatial voting models, including voters who are perfectly informed about candidate platforms. We follow more recent models and relax the assumption of identical voter preference intensities. We call a voter's preference intensity a *strength*, interpreted as the importance that the voter attaches to an issue. Strengths vary, so voters have noncircular indifference curves. Our model departs from standard spatial models in three respects. First, we consider both purely office-seeking parties and ideological parties. Second, our parties are not perfectly informed, and do not know individual voter utility functions. Rather, they obtain information through "test" elections (like opinion polls) that measure how well their current platform would do relative to their opponent's platform. During a campaign, parties test their platforms on voters, receive feedback in the form of vote totals, and alter their platforms to improve vote totals. Finally (as discussed), our parties do not optimize given their limited information. Instead, using the vote totals from test elections, they adap-

tively alter their platforms, trying to defeat their opponent.

Formally, there are two parties competing for  $V$  votes in an  $n$ -dimensional issue space. Each voter's preferences are represented by two vectors of  $n$  integers, which give the voter's ideal positions and strengths on the  $n$  issues. We assume that there are  $k$  possible positions on each issue  $\{0, 1, \dots, k - 1\}$  and  $s$  possible strengths  $\{0, 1, \dots, s - 1\}$ . The utility to a voter from a party's platform,  $y \in \{0, 1, \dots, k - 1\}^n$ , is given by the negative of the squared weighted Euclidean distance, where the weight on the  $i$ th issue is the strength associated with that issue by the voter. If  $s_{ji}$  denotes the  $j$ th voter's strength on the  $i$ th issue and  $x_{ji}$  is the  $j$ th voter's ideal point, then a voter's utility is given by

$$u_j(y) = - \sum_{i=1}^n s_{ji} \cdot (x_{ji} - y_i)^2$$

In this model we assume that both strengths and ideal points are independently and uniformly distributed. The election results we present consider 251 voters, 15 issues, 7 positions per issue, and 3 possible strengths. Therefore, on average, each voter has five issues of major importance ( $s_{ji} = 2$ ), five of minor importance ( $s_{ji} = 1$ ), and five of no importance ( $s_{ji} = 0$ ). The assumption that voter ideal points are uniformly distributed does not necessarily imply regularity. A relatively small number of voters are generated in a large space, so a spray of points is a more appropriate way to think of the distribution.<sup>3</sup> Each voter casts a ballot for the party giving the higher personal utility. To evaluate the trajectory of democratic outcomes, we introduce *centrality*, a measure of the goodness of each outcome. Without such a measure, we cannot compare our model analytically to any other model. We calculate centrality by computing both the sum of the utilities of the individual voters if the winning party were located at the median on all issues and the sum of utilities resulting from the winning party in the election, then divide the former by the latter.

$$c(y) = \frac{\left[ \sum_{j=1}^V u_j(\text{median}) \right]}{\left[ \sum_{j=1}^V u_j(y) \right]}$$

It follows that  $c(\text{median}) = 1$ . This normalization has the interpretation that the higher the centrality, the closer the winning candidate is to the weighted center of voter preferences and therefore the more responsive the democratic outcome. We attach no normative significance to the median itself as an outcome. We merely exploit the fact that it is generally of high aggregate utility. There may exist platforms with centralities greater than 1. Ideally, we would find the platform of minimal average distance and use its utility as the numerator; but the costs in computer time outweigh any advantages. Regardless

of the numerator, we have a measure of aggregate utility, or the average weighted distance to a voter.

Parties are initially represented by a randomly selected "ideal" platform. Since parties are aggregates of individuals, we assume they have uniform strengths across all issues. We consider two types of parties: *ambitious* and *ideological*. Ambitious parties care only about winning elections, and their ideal platform serves only as a starting point for the initial campaign. Ideological parties also want to win the elections, but they want to win with a platform that is spatially close to their ideal platform. Formally, if  $v(y:x)$  is the number of votes the challenger party receives if it takes platform  $y$  and the incumbent is at  $x$ , and  $Y$  is the challenger party's ideal point, then the objective functions for the ambitious and ideological parties can be written as:

$$\text{Obj}_{\text{am}}(y) = v(y:x) \quad \text{if } v(y, x) \leq V/2$$

$$\text{Obj}_{\text{id}}(y) = V/2 + n \cdot k^2 - \sum_{i=1}^n (y_i - Y_i)^2 \quad \text{if } v(y:x) > V/2.$$

Recall that  $V$  is the total number of voters,  $n$  the number of issues, and  $k$  the number of positions per issue. This implies that

$$n \cdot k^2 \geq \sum_{i=1}^n (y_i - Y_i)^2$$

for any  $y$  and  $Y$ . Thus, ambitious parties attempt to maximize votes in the hopes that a larger margin of victory makes them more difficult to defeat in subsequent elections, while ideological parties have lexicographic preferences. Their primary goal is to win the election:  $v(y:x) > V/2$ . Once this is accomplished, they attempt to get as close to their ideal platform as possible.<sup>4</sup>

During each election, the incumbent party's platform is fixed, and the challenger party attempts to find a platform in the issue space that defeats the incumbent. In the first election the incumbent party (arbitrarily chosen) remains at its ideal platform. Thereafter, the incumbent remains at the platform where it won the previous election. The challenger party attempts to defeat the incumbent by choosing a new platform. The challenger party, during a finite campaign, tests new platforms on voters who are assumed to have perfect information about both platforms. These tests are accurate polls of political popularity.

Both ambitious and ideological parties are constrained in how they search the issue space for good platforms. First, the campaigns are of finite length, so parties are limited in the number of polls they can take. For example, a party may only be able to take 40 polls before the election. Second, during any platform adaptation, our parties are limited by the number of issues they can change and the degree of change on any such issue.<sup>5</sup> Even for ambitious par-

ties, the ideal platform functions as an ideological tether in early elections.

Positioning constraints and finite campaign length imply that our parties fail to fulfill their goals optimally. Ideological challengers should typically win elections because ideological incumbents attempt to stay near their ideal platforms, which generally lie in regions of average centrality. However, because of their limited ability and information, ideological challengers accept platforms further from their ideal platforms than necessary. With ambitious parties, after a few elections the incumbents' platforms are located in regions of high centrality, making them difficult to defeat. Therefore, ambitious challengers often lose.

## HOW PARTIES FIND PLATFORMS

Once we relax the assumptions that parties have complete information and the ability to locate optimal platforms, we can model our parties in a variety of ways. There are many ways to not be perfectly rational. We chose three types of parties, each having a different platform search procedure: *random adaptive parties* (RAPs), *climbing adaptive parties* (CAPs), and *genetic adaptive parties* (GAPs). The search procedures were constructed to be crude approximations of actual procedures. More important, they provide reasonable bounds on the ability of parties to locate platforms. The procedures themselves are mechanisms for the party to choose the platform it presents to the voters against the incumbent. All three procedures will be discussed primarily within the context of ambitious parties. The extension to ideological parties is straightforward.

The RAPs are the least adaptive of our parties. Letting  $L$  represent the length of the campaign, RAPs randomly generate  $L$  platforms in the neighborhood of their previous platform and choose the platform that receives the most votes against the incumbent. The RAPs approximate a smoke-filled-room selection process. The party gathers immutable potential candidates and selects the highest vote-getter to represent the party in the election. We do not dispute the contention that this underestimates the ability of parties or candidates to adjust to public opinion. The RAPs are intended as lower bounds on the ability of parties to position themselves.

In contrast to the RAPs, both the CAPs and GAPs selectively refine their platforms to improve vote totals. The CAPs begin with their current platform and experiment, slightly changing positions on a few issues. If the new platform fares better against the incumbent than did the previous one, the party switches to the new platform. These platform tests are called hill-climbing iterations. Here, the campaign length  $L$  equals the number of hill-climbing iterations (including those resulting in no improvement) that a party performs before the actual election. The CAPs enter the election with their final—and therefore best-to-date—platform. The CAPs represent parties

that select a candidate and then adapt the candidate's platform to the electorate's views by testing alterations with focus groups and speeches. After finitely many refinements, the improved challenger faces the incumbent.

The GAPs, the third type of parties we consider, employ a genetic algorithm to guide their search (see Goldberg 1989; Holland 1975). Instead of adapting a single position, genetic algorithms adapt a population of platforms, attempting to discover nonlinear (epistatic) interactions among variables. Genetic algorithms were designed to work well in "complex" environments—spaces with nonlinearities, discontinuities, noise, and high dimensionality. For our purposes, GAPs provide an indication vis-à-vis the other algorithms of the inherent difficulty of the spatial election problem.

The GAPs represent parties whose potential candidates shift positions both by borrowing from competitors and by testing their own alterations. The genetic algorithm generates new platforms using three procedures. It begins with the random creation of, say, 12 platforms. The first operator, reproduction, randomly selects (with replacement) 12 pairs of candidates from the list and reproduces only the preferred member of the pair. The resulting candidates are then randomly arranged in pairs to which the cross over operator is applied. During crossover, the candidates randomly decide (with probability 50%) whether or not to trade positions on a few issues. If they decide to switch, they exchange groups of positions. Finally, the mutation operator allows each candidate to alter positions randomly on an issue or two.

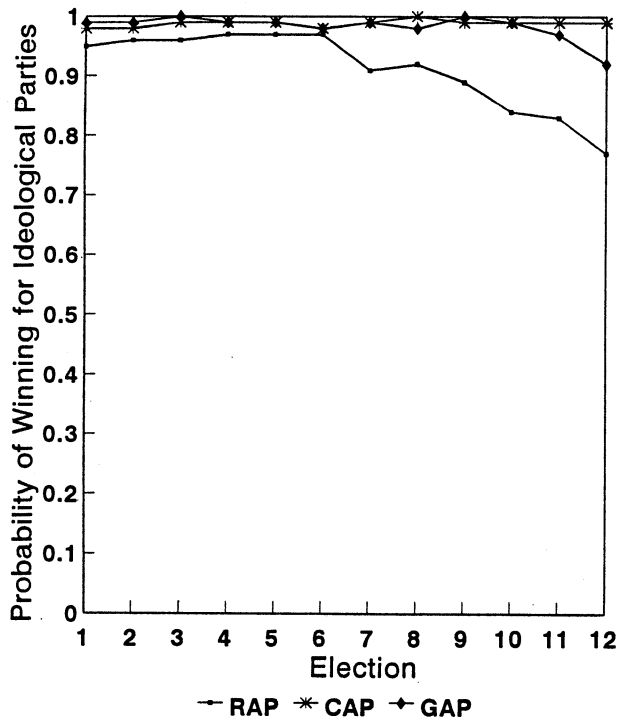
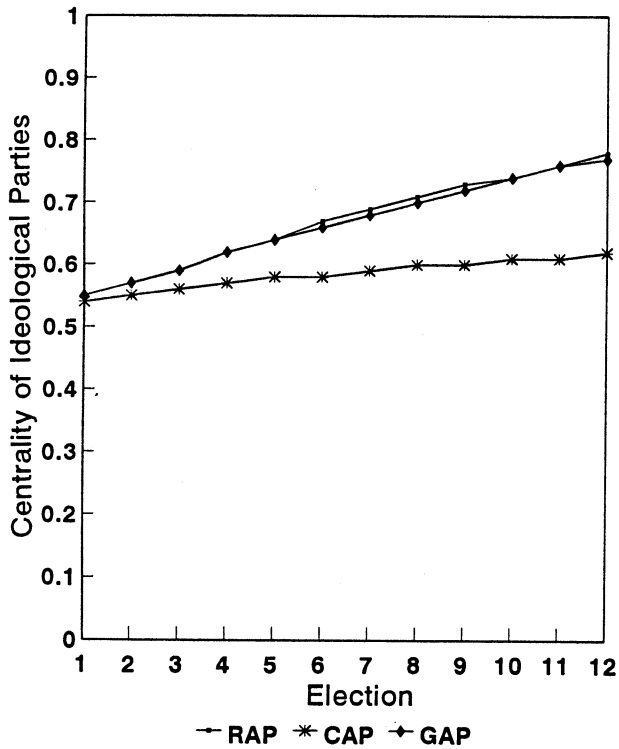
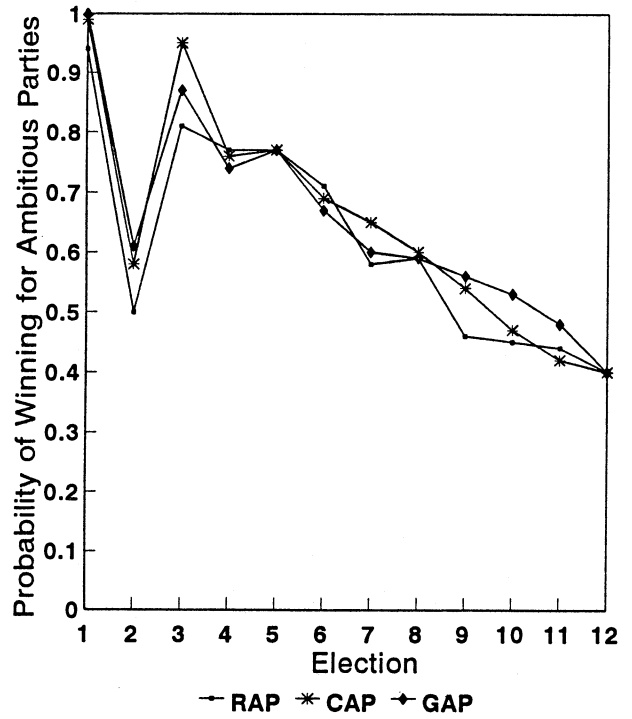
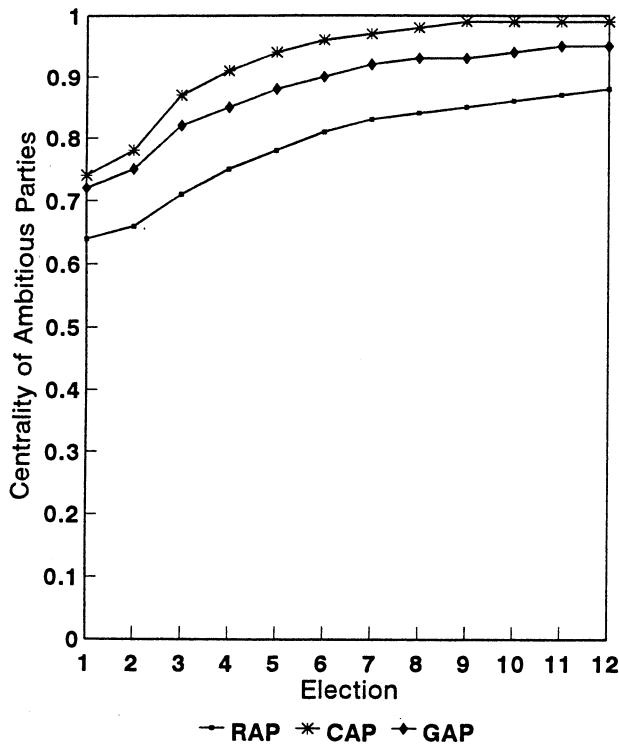
Following biological convention, each application of the reproduction, crossover, and mutation operators is called a *generation*. We count each generation of the genetic algorithm as two units of campaign length, since both crossover and mutation involve candidate platform alterations. At the completion of a campaign of length  $L$ , which consists of  $L/2$  generations of the genetic algorithm, the party chooses the best-to-date platform.<sup>6</sup>

## RESULTS

Several measures are of interest in comparing the different parties and search procedures. Given that our primary concern is the extent to which the distribution of winning party platforms is biased toward regions of high centrality, we record the centrality of winning platforms. We also want to measure the ability of ambitious and ideological parties to defeat the incumbent and to know how far ideological parties must stray from their ideal platforms to do so. Finally, we are concerned with the effect of varying the length of campaigns. The length of campaigns corresponds roughly to the amount of information parties have about voters before an election. Campaigns of length 40 ought to enable parties to compete more effectively against incumbents than campaigns of length 5.

**FIGURE 1**

**Centrality and Success of Adaptive Parties**



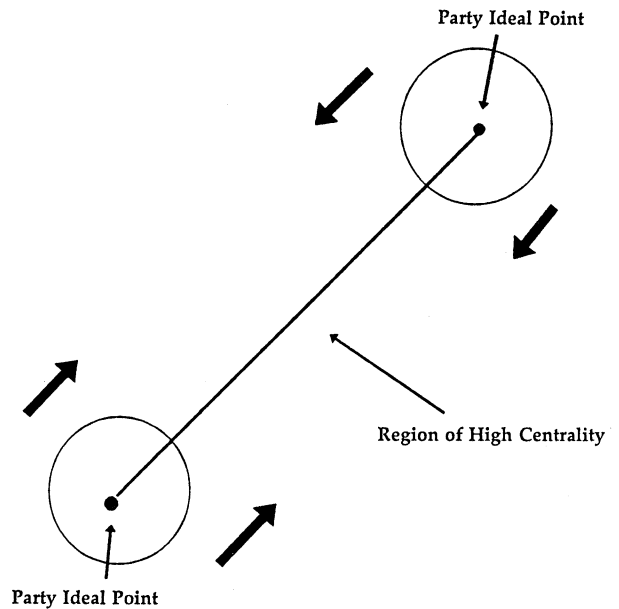
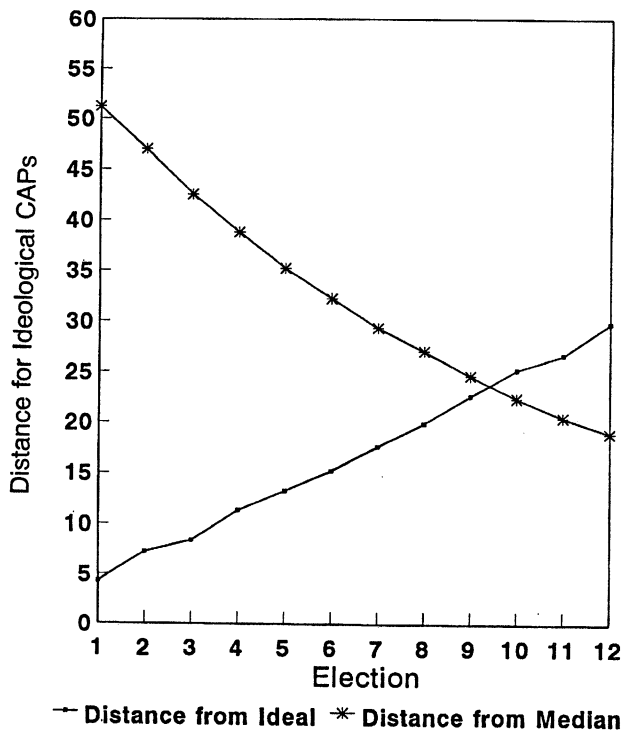
Data represent means over 200 trials

Before presenting our findings, we should note that the robustness of computer simulation results often is sensitive to parameter values. Our findings

appear qualitatively invariant to reasonable changes in parameters. We chose parameter values that seem realistic in the study of democratic elections and for

FIGURE 2

## Ideological Parties: The Dumbbell Waltz



which a wide range of surrounding values produced similar results. For example, we chose 12 elections because most interesting phenomena were manifest by that time. The following parameters were used:

Voter types ( $V$ )	251
Number of issues ( $n$ )	15
Positions per issue ( $k$ )	7
Strengths ( $s_{ji}$ )	3
Elections	12
Campaign length ( $L$ )	40

All of these fall safely within ranges for which we observed no significant changes in the conclusions.

Centrality values have greater significance when viewed with respect to the distribution of centrality. The Appendix shows numerically estimated distribution and density functions for the centrality of platforms. We compare election outcomes to the cumulative distribution function. Note that a winning party with a platform having a centrality of .55 lies in the upper 17% of the distribution. We ran two hundred trials of a 12-election sequence for each party and type of algorithm. We recorded the centrality of the winning party, the probability that the incumbent would be defeated, and (in the ideological case) the distance to the party's ideal platform.

### Ambitious Parties

For all three types of algorithms, ambitious parties moved in directions of higher centrality (see Figure

1). By the sixth election the CAPs and GAPs had average centralities above .9, and the RAPs above .8, which placed all three types of parties in the top .01% of all platforms! It appears that convergence to high centrality and the increase in centrality over time are invariant to the type of search algorithm. As might be expected, the CAPs and GAPs had higher centrality than the RAPs. With respect to the distribution of centrality, however, the differences were not large.

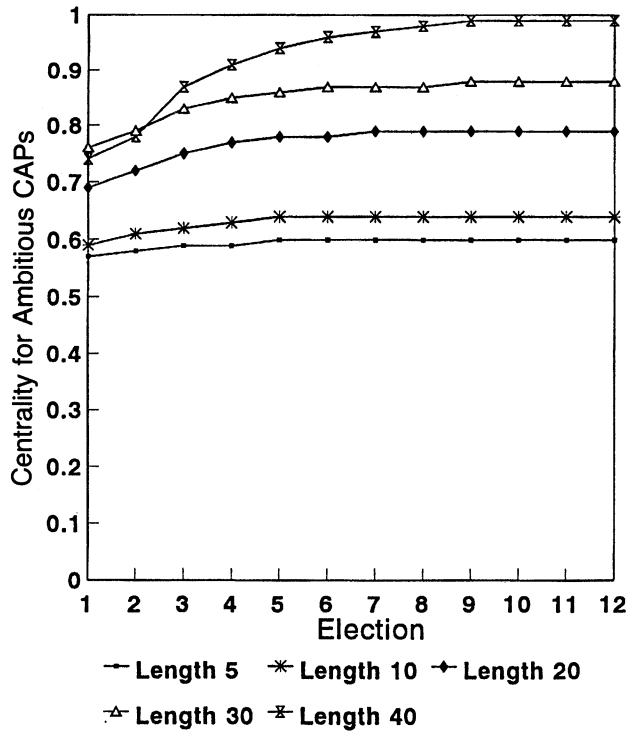
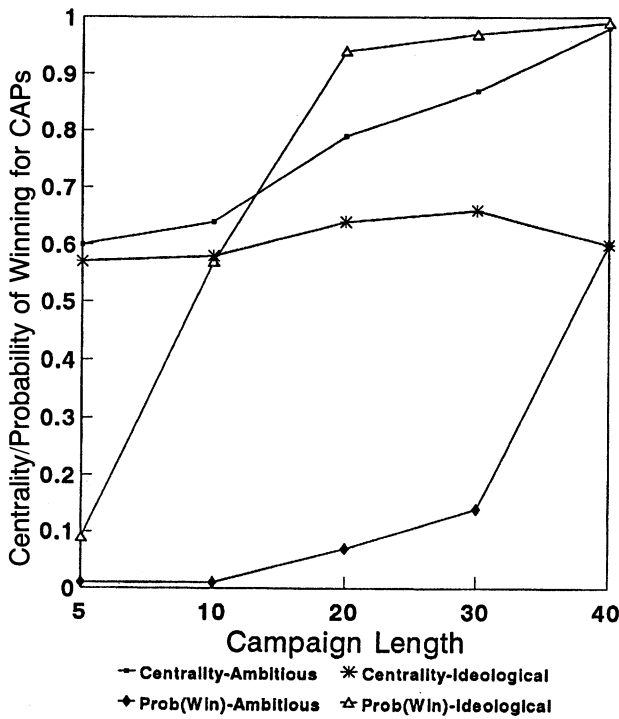
The probability of a challenger's winning decreased from almost 1 in the first election to near .4 by the twelfth election for all three algorithms (see Figure 1).<sup>7</sup> In our model, ambitious challengers have increasing difficulty defeating the incumbent. Incumbency advantage can be attributed to challengers' lack of information, the limitations of adaptive search, and the positions of incumbents' previously adapted platforms.

### Ideological Parties

We would expect the centrality of outcomes in elections between ideological parties to be lower than if the parties were solely ambitious. As a consequence, the probability of winning should be much higher in ideological contests. Our results confirmed these expectations. Centralities were lower overall for ideological parties compared with ambitious parties, and there was less variation over elections (Figure 1). Still, all three algorithms were in the top 3% of platforms at the end of 12 elections. Moreover, the probability of

**FIGURE 3**

**Centrality and Success by Campaign Length**



200 trials  
Values for the 8th election

200 trials

winning was high throughout the sequence of elections, tapering off in the later elections only for the RAPs (Figure 1). Note that the CAPs were not only more likely to win than the RAPs and the GAPs, but they also won with lower centrality. Data reveal the CAPs stayed closer to their ideal points. The CAPs ability to fine tune platforms issue-by-issue generates these results.

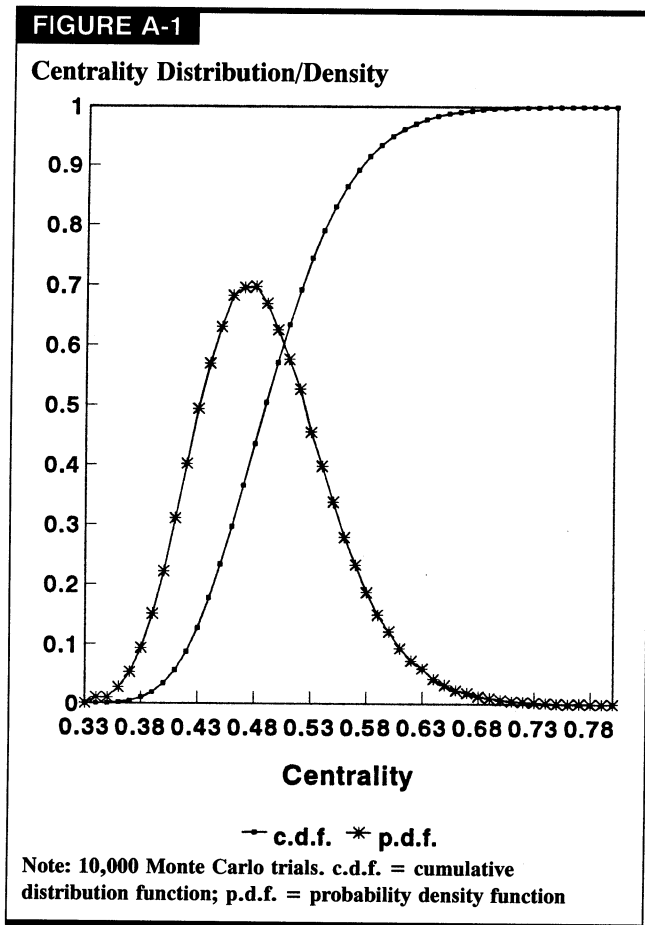
Another noteworthy result came from the simulation of ideological parties. We observed that for all three algorithms, the distance to party ideals increased by small amounts, while the distance to the median decreased over time (Figure 2). We refer to this positioning behavior as the *dumbbell waltz*. The challenging party dances in a neighborhood of its ideal platform until it finds a winning platform. This neighborhood slowly converges to areas of high centrality. A chart of winning platforms would consist of two disjoint neighborhoods (one near each of the ideal platforms) and would resemble a dumbbell. The ends of the dumbbell slowly converge as the number of elections increases (Figure 2).

**Campaign Length**

Increasing the length of the campaign increases the ability of parties to learn about, and adapt to, voters' collective preferences. Figure 3 shows centralities and

probabilities of winning for ambitious and ideological CAPs as the length of the campaign increases from 5 to 40. As expected, both centrality and probability of winning tend to increase with campaign length. To give a complete 12-election example, the figure also reveals how increasing campaign length for ambitious CAPs qualitatively increases centrality. For ideological parties, however, centrality varies only slightly with campaign length (see Figure 3) because more informed and intelligent ideological parties are able to locate winning platforms nearer their ideal platforms.

Our results tend to support the idea that boundedly rational parties will converge to central regions of the issue space in a Downsian fashion.<sup>8</sup> Since centrality measures the closeness of parties to voters' ideal points, winning parties with very high centralities can be said to give voters high utility. In this sense, our results suggest that even with fairly simple adaptive parties (e.g., the RAPs), a two-party system should lead to normatively appealing outcomes. Not surprisingly, ambitious parties reach higher centrality than ideological parties. Finally, all three search procedures for both types of parties produced similar outcomes, indicating that there may exist large equivalence classes of adaptive behavior by parties that may allow researchers to undertake a unified analysis. Substantively, our use of three algorithms bol-



sters the conclusion that departures from the strict informational and rationality assumptions of earlier formal models lead us to convergence results.<sup>9</sup>

## DISCUSSION

Rational choice theories of electoral competition have led to two incompatible conclusions. On the one hand, convergence results offer an intuitively appealing explanation for the responsiveness and stability of two-party systems. On the other, chaotic results raise doubts about the legitimacy of democratic outcomes. Both sets of results have relied on strict assumptions of complete information and global optimization. We have relaxed these assumptions about parties and have observed the parties in our model (even ideological ones) converging to central regions of the issue space. Although some mathematical models indicate that democracy might yield arbitrary results with respect to voters' preferences, our model suggests that this typically will not happen with adaptive parties. Moreover, boundedly rational parties may not be able to defeat incumbents. Mathematical proofs of the existence of winning strategies should not be equated with the ability of human actors or organizations to locate them.<sup>10</sup>

Our results, supported by their invariance to pa-

rameter choices and search procedures, also help confirm basic intuitions about the role of information and ideologies in elections. As the length of campaigns increases or as parties have more information about voters, parties tend to converge toward centrist outcomes. This observation seems to be consistent with general comparisons of both national and local elections. Extreme candidates rarely emerge as national candidates; and when they do, they lose by a wide margin. Yet at the local level, extremists can thrive. The lack of information among voters has received a great deal of theoretical attention; the lack of information among parties and candidates deserves to be explored, as well.

Our techniques are designed to analyze the behavior of a complex adaptive system, a system in which endogenous aggregate behavior emerges from the knowledge- and rule-based behavior of individual agents. Our model can be extended in many directions, incorporating a variety of components known to exist in the real world. It is possible to include correlated preferences, noisy polls, coadaptive parties, interest groups, issue polling, voters with incomplete information, and multiple parties in future models.<sup>11</sup> With all of these extensions, there is the opportunity to calibrate a simulation to data.

## APPENDIX

Figure A-1 shows numerically estimated distribution and density functions for the centrality of platforms.

## Notes

Complete data and computer programs are available upon request. We wish to thank Jeffrey Banks, Michael Kirschenheiter, Roger Myerson, Benjamin Page, and Stan Reiter for helpful comments on earlier drafts. The research support of the Center for Mathematical Studies in Economics and Management Science at Northwestern University and the Santa Fe Institute and a grant from Sun Microsystems are gratefully acknowledged.

1. The top cycle set is the smallest set of points whose members defeat all points not in the set. For example, if A defeats B, B defeats C, C defeats A, and each defeats all other points in the space, then {A, B, C} is the top cycle set.

2. By *logical consistency* we mean in the narrow sense of adherence to a rule and not in the broader sense of satisfying mathematical axioms.

3. Central limit theorems and the like are not appropriate given the relatively small number of voters and the size of the space. With 15 issues, 7 positions, and 3 strengths, there are  $21^{15}$  ( $\approx 10^{20}$ ) possible voter types.

4. This construction allows an ideological party to choose a platform that is further from its ideal platform than the incumbent's platform is. However, this only occurs when the distance between party ideal points is improbably small.

5. As an example, in a five-issue, seven-position space, where the current party platform is (6, 2, 5, 4, 1), the party will be much more likely to test the platform (5, 2, 5, 4, 2) than the platform (1, 6, 1, 3, 6), given their relative distances to the current platform.

6. We use a crossover rate of .5; so on average six platform adaptations occur per generation. Therefore, in  $L/2$  generations, the GAPs will test more than  $L$  platforms.



7. The dip in the second election results from our random assignment of an incumbent prior to the first election. Half the time, the challenger defeats the incumbent prior to adapting a new platform. After adaptation, the election is a landslide. Subsequently, the incumbent (now the challenger) has a difficult time winning the second election against such a strong opponent.

8. Kramer (1977), relying on stricter assumptions, has shown that parties will converge toward the minmax set (the set of platforms that lose by a minimal amount) but that the minmax set is not a stable attractor. In other words, parties can leave the minmax set to win once they converge to it. Our results suggest that if movements out of the minmax set occur, they do not, on average, lower centrality.

9. The equivalent performances of the GAPs and the other less sophisticated algorithms also suggest that the underlying search problem is relatively easy.

10. In chess, a finite perfect information game, an optimal strategy exists, yet no one has found it to date.

11. Convergence tends to be robust to changes in the distribution of voters' preferences, although convergence rates may change. For a complete discussion, see Page, Kollman, and Miller (1992).

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