An asymmetric information model of rumor spreading

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Abstract. We are modeling the scenario where a rumor has broken out in a population. The populations consists of people who want to keep the rumor alive, people who do not care about it and people who want to quash it. We also assume that individuals do not know others' preferences (asymmetric information).

1 The model

The population is composed of 5 types of agents:

1. type $A1$, know the rumor and want to spread it (Arsonist-with-rumor).
2. type $A2$, do not know the rumor. If they do, they want to spread it (Arsonist-without-rumor).
3. type P, passive people who do not care about the rumor.
4. type $B1$, know the rumor and want to quash it (Thwarters-with-rumor).
5. type $B2$, do not know the rumor. If they do, they want to quash it (Thwarters-without-rumor).

Whenever a person who knows the rumor meets a person who does not know the rumor, he tells him the rumor. People of type $A1$ will try to convince passive people of the rumor and will succeed with a probability C_a. People of type B1 will try to convince type $A1$ people to become passive; they will succeed with a probability C_b. The convincing act happens only if the people know the rumor.
The model is initialized with a single agent who knows the rumor and wants to spread it. The rest of N-1 agents consist of A_4^i, P^i, B_4^i, where i stands for *initial*.

The interactions among these people are summarized in the following matrix:

Fig. 1. The interaction matrix

	A_1	A_2	P	B_2	B_1
A_1	-	A_1/A_1	$\frac{C_A \rightarrow A_1/A_1}{1-C_A \rightarrow A_1/P}$	A_1/B_1	$\frac{C_B \rightarrow P/B_1}{1-C_A \rightarrow A_1/B_1}$
A_2	A_1/A_1	-	-	-	A_1/B_1
P	$\frac{C_A \rightarrow A_1/A_1}{1-C_A \rightarrow A_1/P}$	-	-	-	-
B_2	A_1/B_1	-	-	-	B_1/B_1
B_1	$\frac{C_B \rightarrow P/B_1}{1-C_A \rightarrow A_1/B_1}$	A_1/B_1	-	B_1/B_1	-

We decided to run this model on three different network topologies:

1. a fully connected network (*mean-field*)
2. an Erdös-Renyi random network
3. a scale free network formed by preferential attachment.

2 Results

2.1 Fully connected network: the baseline scenario

This is the "mean-field picture" and so we can describe it with the following set of master equations (Krapivsky et al., 2010):

$$\dot{a}_0 = -C_a a_1 a_0 + C_b a_1 a_3 \quad (1)$$

$$\dot{a}_1 = a_1 a_2 + C_a a_1 a_0 - C_b a_1 a_3 + a_2 a_3 \quad (2)$$

$$\dot{a}_2 = -a_1 a_2 - a_2 a_3 \quad (3)$$

$$\dot{a}_3 = a_3 a_4 + a_1 a_4 \quad (4)$$

$$\dot{a}_4 = -a_3 a_4 - a_1 a_4 \quad (5)$$

Where $\dot{a}_0$ is the fraction of passive individuals, $\dot{a}_1$ is the fraction of $A1$ individuals, $\dot{a}_2$ is the fraction of $A2$ individuals, $\dot{a}_3$ is the fraction of $B1$ individuals and $\dot{a}_4$ is the fraction of $B2$ individuals.

We can obtain the steady state solutions to the above equations by setting the derivatives to zero and thus finding:

$$a_0^\infty = \min\left\{ \frac{C_b}{C_a} a_4^i, 1 - a_4^i \right\} \tag{6}$$

$$a_1^\infty = \max\left\{ 1 - a_4^i(1 + \frac{C_b}{C_a}), 0 \right\} \tag{7}$$

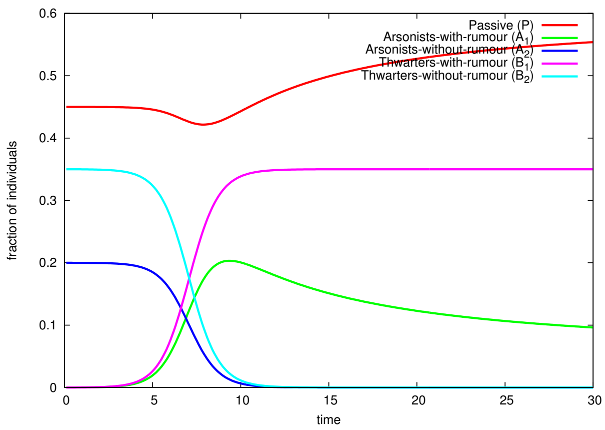
$$a_2^\infty = 0 \tag{8}$$

$$a_3^\infty = a_4^i \tag{9}$$

$$a_4^\infty = 0 \tag{10}$$

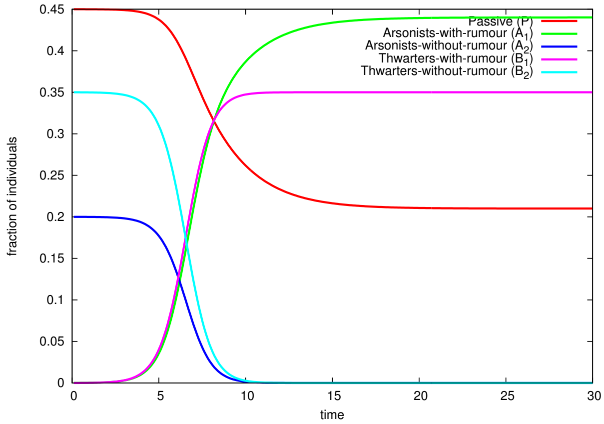
This tells us that the asymptotic fraction of passive agents is proportional to the ratio of convincing abilities of the two types of people and proportional to B_2^i.
To get an idea, let us consider an example with $C_a = 0.3$ and C_b=0.5. We numerically solve the master equations and get the following time-evolution of different fractions of the population.

Fig. 2. Numerical solution for equations (1):(5) with $C_a = 0.3, C_b = 0.5$



The solution suggests that there is a complex interplay between the "active" rumor spreading and the quashing players in swinging the passive population. Indeed, the quashers are more successful given that $C_b > C_a$. We considered also the opposite scenario where $C_a > C_b$. We find that expectedly the spreaders have a larger asymptotic fraction.

Fig. 3. Numerical solution for equations (1):(5) with $C_a = 0.5, C_b = 0.3$

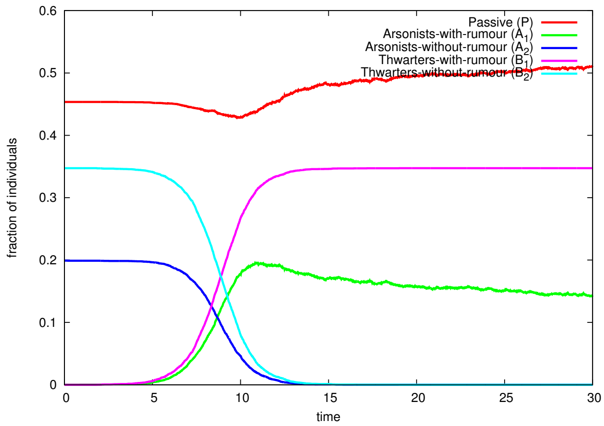


However, real world networks are not fully connected and more importantly, they are sparse with a finite average degree. We decided to proceed by steps by gradually adding more realistic features to our model. We start by considering the Erdös-Renyi random network.

2.2 Erdös-Renyi random network: the second scenario

In this scenario, we simulate this model on an Erdös-Renyi random network with size of the population $N = 10,000$ and an average degree $\langle k \rangle = 20$. We used the same parameters (probability to succeed in convincing others) of the baseline scenario and we find that the qualitative features of the simulation are preserved.

Fig. 4. Simulation Erdos-Renyi random network with $C_a = 0.3, C_b = 0.5$

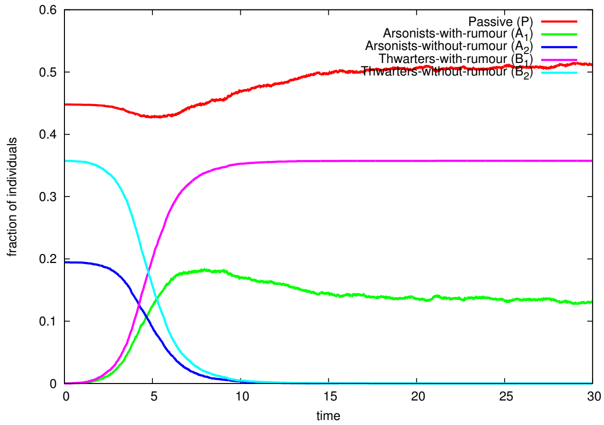


We observe that the asymptotic fractions are the same as in the baseline scenario, but the evolution is slower.
However, real world networks have fat tails in the degree distribution, corresponding to a *power law*.

2.3 Scale free network and Preferential attachment: the third scenario

In this scenario we use Barabsi et al.'s *Preferential attachment algorithm* to generate power law/scale free networks.

Fig. 5. Simulation on scale free network with $C_a = 0.5, C_b = 0.3$



In this case, we observe that the asymptotic fractions are the same as in the baseline scenario, but the evolution is faster.

3 Conclusions

In this paper we describe a simple approach to model the spreading of rumors in a mixed population of people favouring it and others against it.
We show the master equation approach to solve the mean-field case. We then explore the effects of changing the underlying network topology on this model through simulations. We report the results for the cases where (1) the spreaders are stronger and (2) the quashers are stronger in their ability to sway the opinion of the passive population.
The non trivial result that emerges from our model is that, even though interactions are symmentric, the quashers come out stronger than spreaders. Moreover, we observe that the underlying network topology does not affect the asymptotic fraction of different types of agents.

Among the possible extensions of this model, we argue that it can be applied to study the dynamics of the evolution of opinions of voters during times of elections.
Furthermore, it would be interesting to study the effect of network clustering on the model we set up.

Bibliography

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