Where to put your money if you want to spread a rumor

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Abstract

We investigate the situation in which a rumor has broken out and some folks want to keep it alive while others want to quash it. We develop a simple model to study how the rumor spreads in a social network and how it depends on the communication probability of rumor believing agents. We further analyse how knowledge of the underlying structure of the networks affects the effectiveness of campaigns to either promote or quash the rumor.

1 Introduction

In our model a rumor spreads via communication on a social network. Agents in the social network either believe or do not believe in the rumor. An agent's belief in the rumor is a function of its neighbors' beliefs and an exogenous bias that captures the net influence of rumor promoters and quashers. In this project we focus on how two questions:

- What factors determine whether a rumor is promoted or quashed in a network?
- How does knowledge of the structure of social network affect the effectiveness of rumor promoting or quashing campaigns?

2 Model

Consider a population of N agents, which communicate on a social network. The social network is represented by a random undirected graph specified by the adjacency matrix W. The state x of agent i can take one of two values: $x_i = \{-1, 1\}$. A value of 1 corresponds to the case in which the agent believes in the rumor and communicates it to its neighbors. In the opposite case the agent does not believe the rumor and spreads the opposite message to its neighbors. In our model, an agent has no memory of her prior day's belief or disbelief and thus may believe the rumor today but disbelieve it tomorrow.

Via the social network agents receive the following signal:

$$S_i = \frac{\sum_j W_{ij} v_j x_j}{k_i} + \mu_i, \tag{1}$$

where S_i is agent i's signal and k_i the degree of agent i. v_j is a Bernoulli random variable that encapsulates the probability that one of i's neighbors communicates with agent i. μ_i is the net influence of promoters and quashers on agent i. We make this probability state contingent. In particular we have:

$$P(v_j = 1) = \begin{cases} p, & \text{, if } x_j = 1, \\ q, & \text{, if } x_j = -1, \end{cases} \tag{2}$$

where in general $q < p$. This reflects that fact that people are more likely to communicate about news (rumors) rather than non-news.

Agent i then updates its state to $x_i = 1$ with probability:

$$P(x_i = 1) = \frac{1}{1 + e^{-bS_i}}, \tag{3}$$

where b is an intensity of choice parameter. The larger b the more sensitive the agent i will be to the signal. As b decreases the dynamics of the model will become increasingly dominated by the noise. The model is simulated asynchronously, i.e., at a given time a random agent is selected and its state is updated. We initialise the system such that an agent has $x = 1$ with probability p_i.

3 Results

3.1 What factors determine whether a rumor is promoted or quashed in a network?

All results below are generated with Erdos-Renyi networks unless otherwise stated with a population size $N = 100$ and connection probability of 0.04. We begin by illustrating the convergence properties of the model for different exemplary parameter values. For this purpose we plot the average belief over time in figure 1. The behavior of the population is strongly dependent on the initial conditions, in particular how many agents believe the rumor at the start. For $q = p = 1$ we plot the dependence of the final value of the average belief on p_i, see figure 2. It is clear that there is a critical probability of initial rumor spread p_i after which the system tips over into one of the two consensus states. Also in figure 2 we plot the critical probability p_i as a function of the communication probability q in the figure on the right. As the communication probability of non believing agents goes down less and agents have to believe initially in the rumor for it to spread successfully.

3.2 Campaign impact of knowledge of network structure

We now consider the impact of the net bias. In particular we consider the following scenario. Both promoters and quashers have a certain budget B_p and B_q respectively at their disposal to influence the agents' belief. The cost of influencing an agent is fixed at $c = 1$. Promoters influence the population as follows: they pick the 10 agents with the highest degree and invest the entire budget into these agents. Quashers follow a different strategy: they spread their

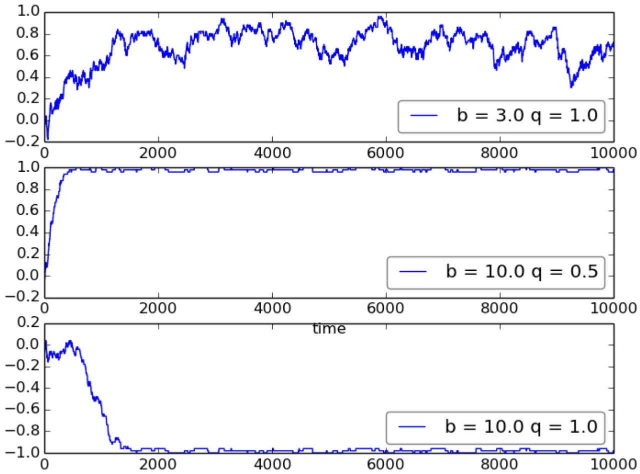

Figure 1: Time series of the average belief for three example parameter values. The top plot illustrates that for low values of b the population does not converge and noise dominates the behavior. The bottom plot shows the opposite: For higher values of b the agent's beliefs are dominated by their signal and the population converges. In the middle plot, if the probability of communication for "quashing" agents is low ($q = 0.5$) the rumor spreads.

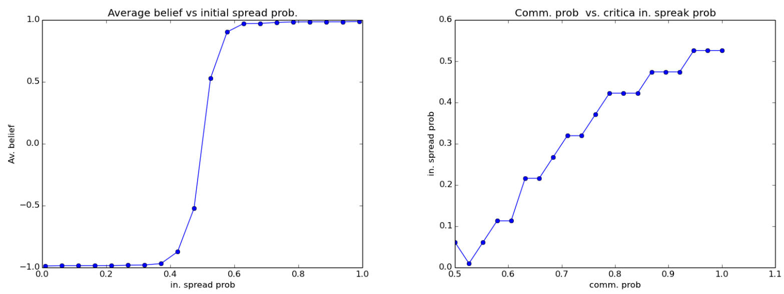

Figure 2: Left: Dependence of the final value of the average belief on p_i. There is a phase transition at $p_{i,C} = 0.5$. Right: We investigate the dependence of the critical initial probability $p_{i,c}$ on the level of q while keeping $p = 1$.

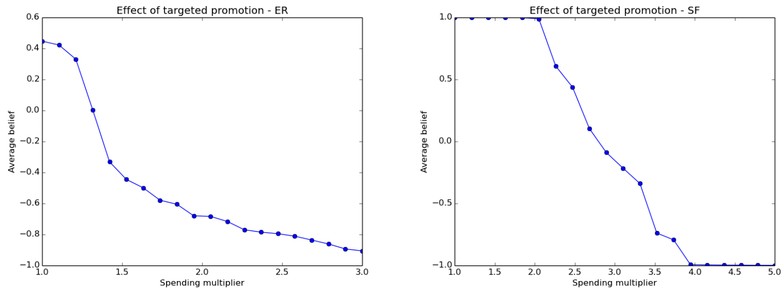

Figure 3: Left: Average belief as a function the ratio B_q/B_p for an ER network. Right: Average belief as a function the ratio B_q/B_p for a preferential attachment network.

budget evenly across all agents. We then have for μ_i:

$$\mu_i = \begin{cases} \frac{B_p}{10c} - \frac{B_q}{Nc} & \text{, if i in top 10 agents,} \\ -\frac{B_q}{Nc} & \text{, otherwise.} \end{cases} \tag{4}$$

We are interested in how much more money the quashers have to pay in order to compensate for their lack of knowledge of the network structure. In figure 3 we plot the average action as a function of the ratio B_q/B_p for an Erdos-Renyi network. On the bottom right we plot the same for a preferential attachment network. In both cases we start from an initial condition in which the population would converge to -1 without the bias. It is clear that the target strategy based on the knowledge of the network structure is a more successful campaign strategy. This effect is even stronger in the preferential attachment network where the degree distribution is more skewed than in the ER network.

4 Summary of Key Results

- As figure 1 shows the level of b influences the behavior when q is held constant. For low values of b the population does not converge and noise dominates the behavior. For higher values of b the agent's beliefs are dominated by their signal and the population converges.
- If the probability of communication for "quashing" agents is low ($q = 0.5$) the rumor spreads, given relatively high values of b.
- Rumor spread depends on a critical probability of initial rumor spread.
- A targeted campaign to promote the rumor is more effective than an untargeted campaign to squash the rumor. Quashers have to spend about 1.5 (2.5) more than the promoters to swing the population's opinion for the Erdos-Renyi (preferential attachment) network.

5 Future Directions and Questions

The context in which the agent receives the signal matters. For example, if the agent receives the signal from a leader of a group in which they belong, they may be more likely to give credence to the rumor. So not only does neighborhood connection matter, the agent's membership in organizations may also matter. There may be a difference between not believing the rumor and then actively stopping the rumor versus taking no action. Factors in the environment may also play a role in rumor dissemination. We modeled one factor, money, but did not model the way money is used. It could be used to buy media or to buy influence with people such as through the bestowing of gifts. The way in which a rumor is communicated may also play a role. If a rumor is communicated through a trusted neighboring agent, it may hold more weight than if the rumor is "heard" on social media.

6 Other Applications

- Spread of disease. The spread of disease can be modelled using the same basic model. Spread of a contagious disease is contingent on coming into contact with the virus. It usually requires person-to-person contact. However, person-to-person contact does not guarantee disease transmission.
- Campaigns for elective office. Campaigns for elective office often involve word-of-mouth information transmission. Voters may rely on what they hear from persons in their social network or physical environment. Campaigns can target certain network hubs to increase the return on campaign dollars.
- Take up. Persons in social networks often take a certain action because someone in their network takes the action. For example, many persons are eligible for Medicaid in the U.S. but do not enroll. When persons they know and respect enroll, it becomes socially acceptable for them to enrol. The actions demonstrated by those in their network influences their own actions.