Homework
A model of regime switch

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1 Introduction

We model decisions to participate in an anti-government demonstration as primarily a problem of communication and coordination. Participation in a demonstration is reflected by the number of people who attend. Agents make choices about likely participation levels by interacting with the people they are linked to. In this model people are made to adapt to others choices in terms of the payoffs. However a certain amount of random behavior is not ruled out of the social interactions. Thus we try to incorporate this in our model and see the effect of this variation on how the aggregate behavior emerges.

2 The model

There is a set of agents of size N. Each agent i chooses to participate ($j_i = 1$) in the demonstration or not ($j_i = 0$). If agents i joins the demonstration, she may be repressed ($r_i = 1$) or not ($r_i = 0$). We set $x = 1$ if the regime is toppled and $y = 0$ otherwise. Letting $M = \sum_{i \in N} j_i$, we assume $Pr(y = 1|M) = \dfrac{\log(M)}{\log(N)}$. This functional form is chosen to make the function concave, but is not special in any other way. The probability of an individual demonstration partipant being repressed is $Pr(r_i = 1|M, j_i = 1) = \frac{1}{J}$.

The payoff to agent i is

$$u_i(j_i, r_i, y) = p_i j_i + I(y = 1) - c \cdot I(r_i = 1)$$

where i is the standard indicator function. The number $p_i \in (0, 1)$, which differs between agents, represents utility gained from participation that does not depend on the outcome. We will refer to this parameter as the level of agent's "motivation" to contribute to the movement. In the simulations we have set $c = 25$. The results depend on c in a predictable way that is obvious from the analytical statement in the next paragraph, so we do not show results for different values of c.

Each agent will form some expectation about the number of people other than herself that plan to participate. This expected turnout by all other agents under i's beliefs will be denoted $\mu(M_{-i})$. The agent joins the demonstration if the expected utility of joining is at least as high as for not joining. Since we will assume that agents' beliefs place full probability mass on a single value of M_{-i}, this is true if

$$p_i - \frac{c}{\mu(M_{-i}) + 1} + \frac{\log(\mu(M_{-i} + 1)}{\log(N)} \geq \frac{\log(\mu(M_{-i}))}{\log(N)}.$$

The learning process occurs in a network. The agents update beliefs about the expected level of participation by communicating their friends in the network. For this simulation, we use a random undirected graph where the probability of any edge is .5. Agents communicate hostely about whether they will join the demonstration and agents expect that the proportion of people participating in the demonstration will be the same as the proportion of their friends who are participating.

Communication works as follows. First, agents start with a (common) initial expectation M_0 about the number of participants and make initial participation decisions after setting $\mu(M_{-i}) = M_0$. Second, agents are ordered randomly and sequentially update expectations and participation decisions based on their local network information. At the end of this process, the demonstration takes place and payoffs are realized. We interpret the starting point M_0 as characterizing the information available to the agents at the beginning of the process. This is a variable that is in principle manipulable by an outside actor. The simulation is run 1,000 times.

In the second model, we also allow p_i to evolve over time. The process for updating p_i involves each agent taking a payoff weighted average of her neighbors. In computing the payoff-weighted averages, we have run the model both using the average payoff over all of the runs and using the most recent payoffs. The results look nearly identical in both cases.

3 Results

The results for the baseline model show that the system reaches equilibrium almost immediately. Furthermore, though the results depend on the value chosen for M_0, the starting values only make a difference in terms of selecting one of two equilibria. Figure 1 shows the dynamics of participation in the baseline model.

The results for the second model, in terms of the number of participants, is similar to the baseline model in that a stable equilibrium is reached almost immediately. However, in the second model, the same low-turnout equilibrium is selected for any starting values. Figure 4 gives the same dynamics for the second model.

A look at how p_i evolves in the model reveals why the low turnout equilibrium is always selected. At the end of the runs, all agents have adjusted all the way to $p_i = 0$. The result is perhaps surprising given that high values of p_i give agents an automatic bonus in their utility function without directly hurting the agent. However, agents adjust to low values of p_i because the most patriotic agents turn out early on and some are repressed. Thus, agents quickly adjust their p_i values downward so that they are unlikely to imitate those who participated and were repressed. Figure **??** shows this result.

4 Conclusions

Our results suggest that the level of information that people start with may shift this system from a low turnout equilibrium to a high turnout equilibrium. Manipulation of information in the past has occurred through distribution of literature or state-sponsored radio broadcasts, so this is a relevant consideration. Furthermore, as the motivation levels evolve and some repression at any given demonstration is very likely, agents coordinate on the low turnout equilibrium almost certainly. However, that in the low turnout equilibrium there is still significant and positive turnout and the probability that the regime topples is above zero.

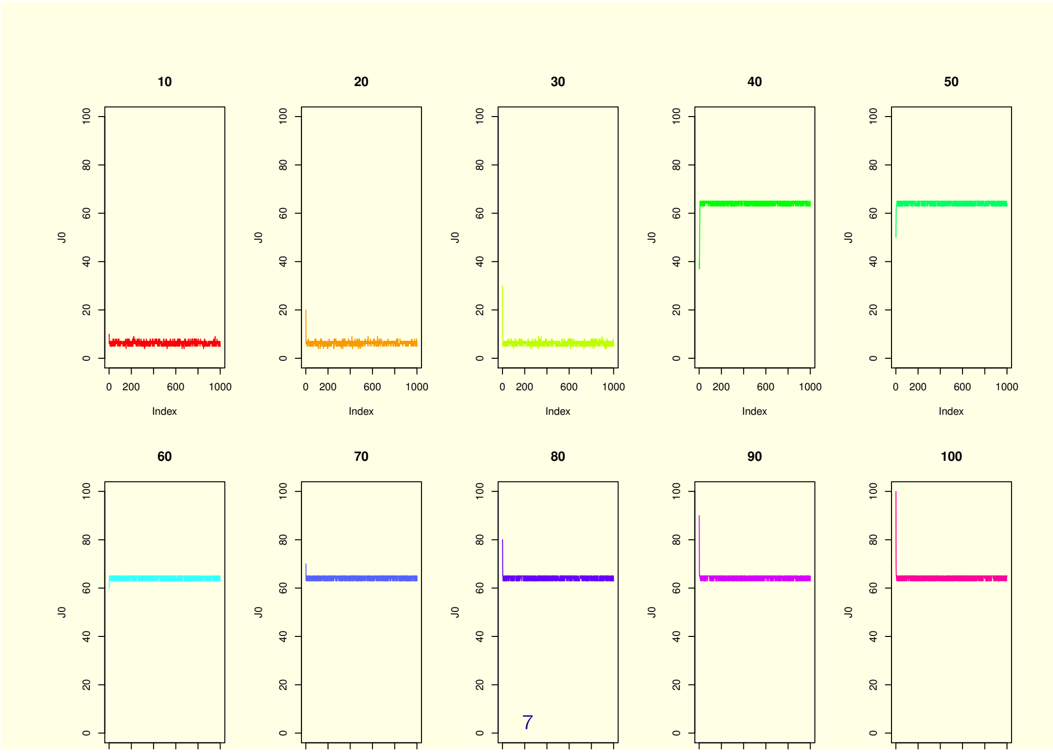

Figure 1: Trace plots of the number of participators over 1000 runs of the baseline model. The number above each plot give the chosen level of M_0.

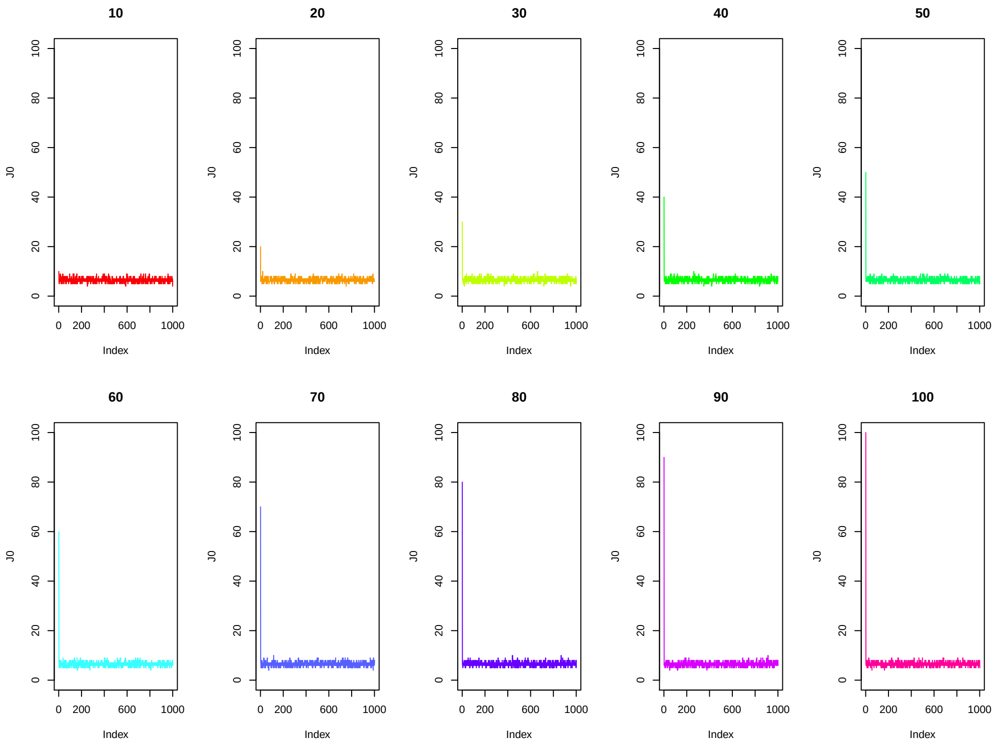

Figure 2: Trace plots of the number of participators over 1000 runs of the endoegnous patriotism model. The number above each plot give the chosen level of M_0.

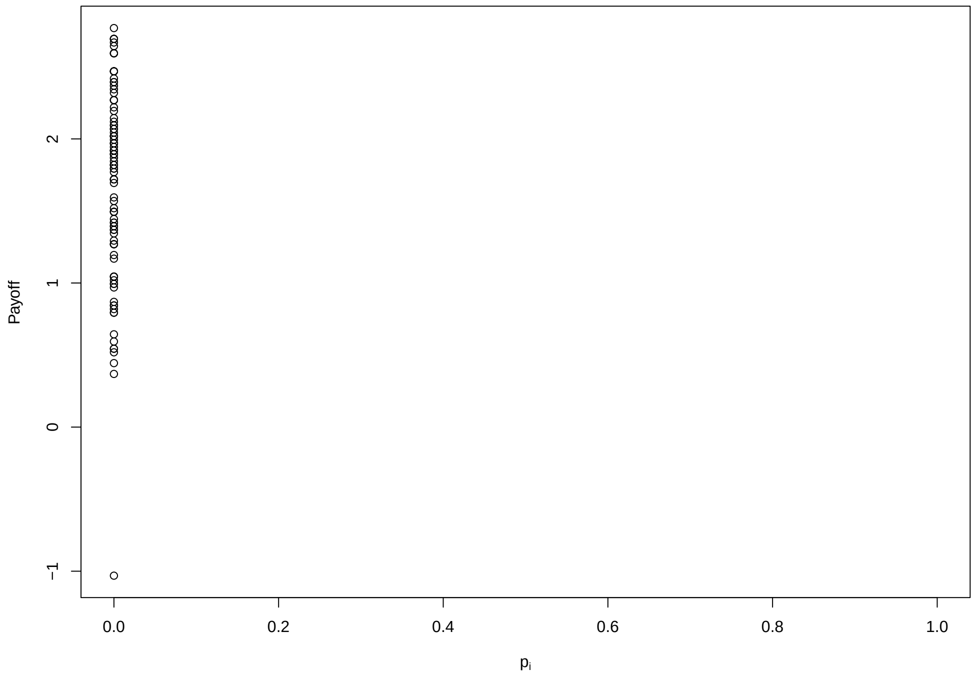

Figure 3: Ending values of p_i for each agent in endogenous patriotism model

5 Appendix: Robustness check

Perhaps the extreme result in the given model is an artifact of the rules for the p_i parameter. To check this, we ran the simulations again where the agents may make some errors in computing the payoff-weighted average of those linked to them, and therefore p_i may fluctuate around the intended value. Errors were drawn from a normal distribution with mean zero and variance σ^2, but truncated at zero and 1. Even in examples with very high variance (σ^2=2 or 5) the equilibrium was not disturbed, though the ending levels of p_i affect the randomness. Figures

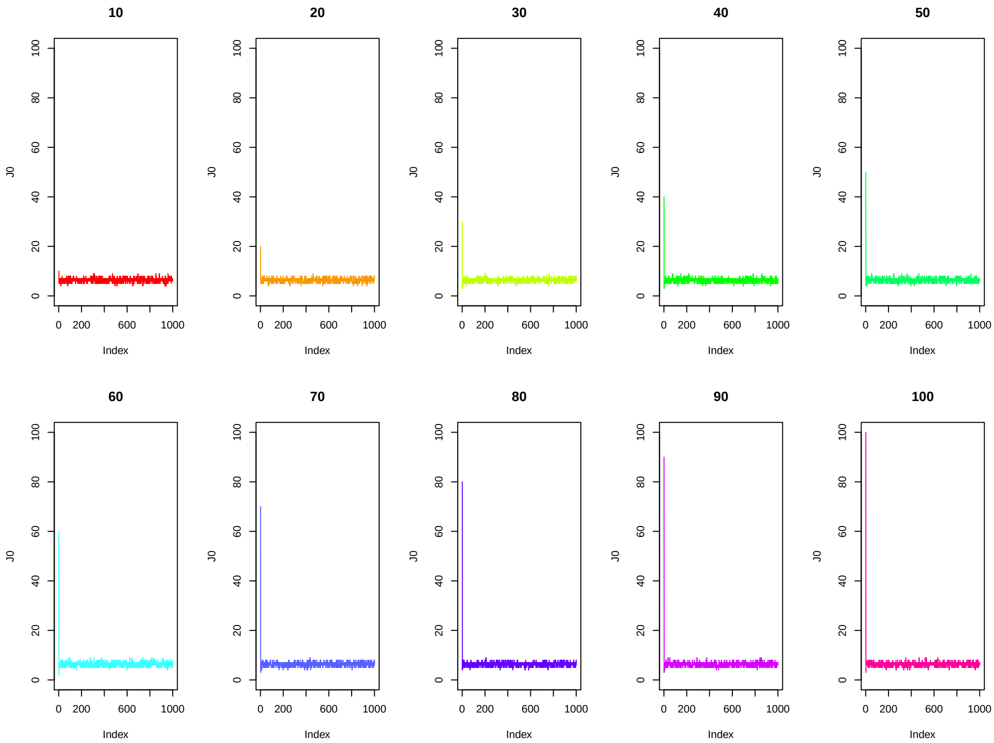

Figure 4: Trace plots of the number of participators over 1000 runs of the endoegnous patriotism model with errors in the updating of p_i.

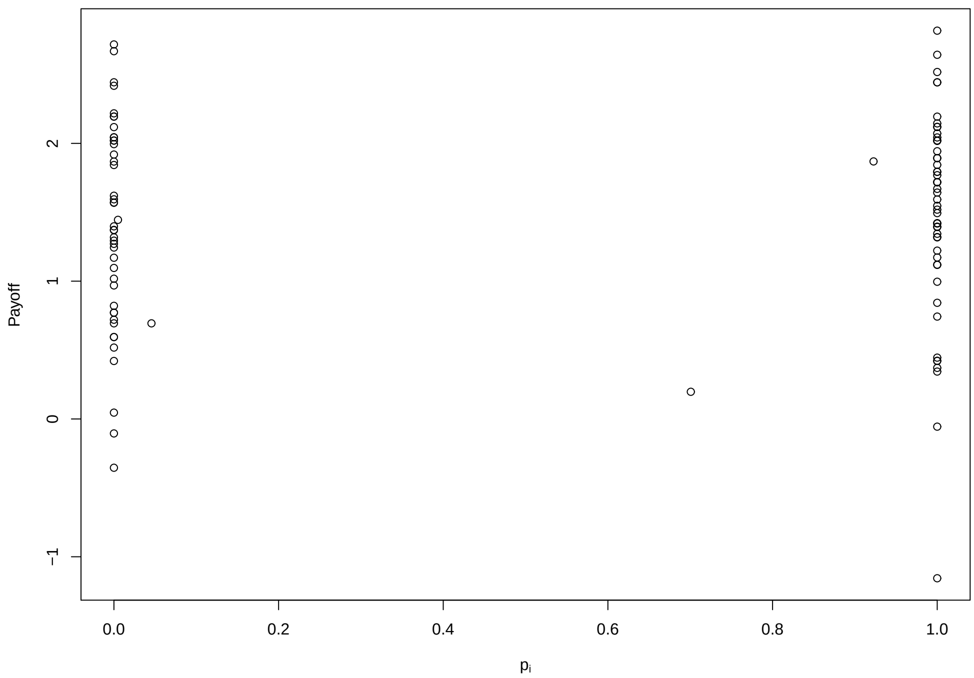

Figure 5: Final values of p_i for agents in the model with stochasticity